21-268 Multidimensional Calculus: Midterm 1.

2017-02-14

- *This is a closed book test. No electronic devices may be used. You may not give or receive assistance.*
- *You have 50 minutes. The exam has a total of 5 questions and 40♡'s.*
- *You may use any result proved in class or any regular homework problem* ***PROVIDED*** *it is* independent *of the problem you want to use the result in. (You must also* ***CLEARLY*** *state the result you are using.)*
- *The questions are roughly in increasing order of difficulty. Good luck, and happy valentines day* ⌣̈.

5♡ 1. Let $f\colon \mathbb{R}^3 \to \mathbb{R}^2$ be defined by

$$f(x) = \begin{pmatrix} x_1 x_2 \sin(x_2 + x_3) \\ |x|^3 \end{pmatrix}.$$

(Recall $x = (x_1, x_2, x_3) \in \mathbb{R}^3$). Find the derivative of f.

5♡ 2. Suppose $f\colon \mathbb{R}^2 \to \mathbb{R}$ and $g\colon \mathbb{R}^2 \to \mathbb{R}^2$ are two differentiable functions such that

$$\begin{aligned} &g(0,1) = (1,2), \quad &&\partial_1 g_1(0,1) = \alpha, \quad &&\partial_2 g_1(0,1) = \beta, \quad &&\partial_1 g_2(0,1) = \gamma, \quad &&\partial_2 g_2(0,1) = \delta, \\ &f(1,2) = 7, \quad &&\partial_1 f(0,1) = a, \quad &&\partial_2 f(0,1) = b, \quad &&\partial_1 f(1,2) = c, \quad &&\partial_2 f(1,2) = d. \end{aligned}$$

(Recall since g is an $\mathbb{R}^2$ valued function, we write $g = (g_1, g_2)$ (i.e. $g(x) = (g_1(x), g_2(x)) \in \mathbb{R}^2$.) Compute $\partial_1 (f \circ g)$ at the point $(0,1)$. Express your answer in terms of only α, β, γ, δ, a, b, c and d.

10♡ 3. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = 3x^2 - 2y$. Show directly using the ε-δ definition that f is continuous at the point $(1,2)$.

10♡ 4. True or false:

If $f\colon \mathbb{R}^2 \to \mathbb{R}$ is differentiable, then it is necessarily continuous.

Prove it, or find a counter example. [This was a question on your homework. Please provide a complete proof (or counterexample here) without relying on this particular result from the homework.]

10♡ 5. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x) = \frac{x_1 x_2}{|x|} \quad \text{when } x \neq 0, \quad \text{and} \quad f(0,0) = 0.$$

(Recall $x = (x_1, x_2) \in \mathbb{R}^2$.) At what points in $\mathbb{R}^2$ is $\partial_1 f$ defined? Compute $\partial_1 f$ at these points. Moreover, determine at which of these points $\partial_1 f$ is continuous. Prove your answer. [When proving continuity, you may use standard theorems and do *not* have to limit yourself to only using the ε-δ definition.]