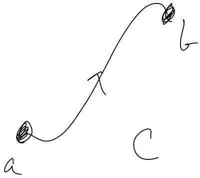
Fundamental Thm of Line Integrals (last time).

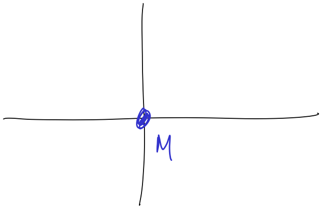
C a C^1 curve
a, b end pts.



$U \supseteq C$ open & $\varphi : U \to \mathbb{R}$ C^1

$$\int_C \nabla\varphi \cdot d\ell = \varphi(b) - \varphi(a).$$

Consequence in Physics: Gravity.

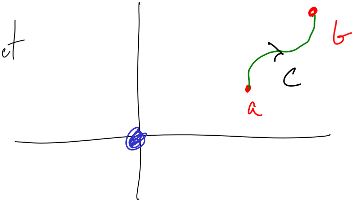


Mass M at origin.
Mass m a $x \in \mathbb{R}^d$.

Force on m is $-\frac{GMm}{|x|^2}\frac{x}{|x|} = F(x)$.

$$F(x) = -\frac{GMm}{|x|^3}x \quad (G>0 \text{ is a (gravitational) universal constant}).$$

Let W = work done against gravity pushing an object from a to b along C



Compute: $W = \int_C -F \cdot dl$

$$\varphi(x) = \frac{GMm}{|x|}.$$

$$F = -\frac{GMm\,x}{|x|^3} = +GMm\,\nabla\left(\frac{1}{|x|}\right) = +\nabla\varphi(x)$$

$$\left(\text{Check: } \nabla\left(\frac{1}{|x|}\right) = -\frac{1}{|x|^2}\nabla|x| = -\frac{1}{|x|^2}\frac{x}{|x|}\right)$$

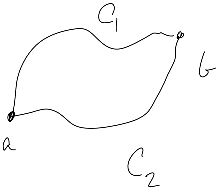
Fund thm: $W = -\int_C F \cdot dl = -\int_C \nabla\varphi \cdot dl = -\varphi(b) + \varphi(a)$

$$= \frac{GMm}{|a|} - \frac{GMm}{|b|}.$$

Potential forces: We say $F : \mathbb{R}^d \to \mathbb{R}^d$ is a potential force if $\exists$ C^1 fn $\varphi : \mathbb{R}^d \to \mathbb{R}$ $\ni$ $F = -\nabla\varphi$

Conservative Force: We say F is a conservative force if whenever C_1 & C_2 are any two curves with the same end points (going in the same direction)

$$\int_{C_1} F \cdot dl = \int_{C_2} F \cdot dl.$$



Above: ① Gravity is a potential force.

② Fund thm $\Longrightarrow$ any potential force is conservative.

③ Question: Is any conservative force a potential force? (IOU).