

21-268 Multidimensional Calculus: Midterm 1.

Wed 09/30

- This is a closed book test. No calculators or computational aids are allowed.
- You have 50 minutes. The exam has a total of 5 questions and 50 points.
- You may use any result from class or homework **PROVIDED** it is independent of the problem you want to use the result in. (You must also **CLEARLY** state the result you are using.)
- The first two questions are easier than the rest. The last three are comparable in difficulty; though depending on your intuition you may find one easier than the other.

5 1. Let $f(x, y) = \begin{pmatrix} \sin(x) \sin(y) \\ \cos(x^2 + y^2) \end{pmatrix}$. Compute Df .

10 2. Let $f(h, \theta)$ be a differentiable function such that

$$\partial_h f = 2h\theta \quad \text{and} \quad \partial_\theta f = h^2 - 1.$$

Let $g(x, y) = f(xy, x^2 - y^2)$. Compute $\partial_x g$ in terms of x and y . (Your answer should not involve f .)

3. Let $f(x, y) = \frac{x^2 y}{x^4 + y^2}$ for $(x, y) \neq (0, 0)$.

5 (a) Let $(u, v) \in \mathbb{R}^2$ and $g(t) = f(0 + tu, 0 + tv)$. Does $\lim_{t \rightarrow 0} g(t)$ exist? If yes compute it.

5 (b) Does $\lim_{(x, y) \rightarrow 0} f(x, y)$ exist? Prove your answer.

10 4. Let $f: \mathbb{R}^m \rightarrow \mathbb{R}^n$ be a function. If for every $i \in \{1, \dots, n\}$ we have $\lim_{x \rightarrow a} f_i(x) = \ell_i$, then show directly using the ε - δ definition that $\lim_{x \rightarrow a} f(x) = \ell$. [This was a question on your homework. Please provide a complete proof here, instead of just citing the appropriate question.]

5. Let $f(x, y) = xy^{1/3}$.

5 (a) If $y > 0$, is f differentiable at (x, y) ? Prove it. [You don't have to do this explicitly from the limit definition, and can use without proof theorems from class.]

10 (b) Is f differentiable at $(0, 0)$? Prove it.