

Concepts of Integration
Tuesday, June 27

1. Let

$$K = \int_3^{16} e^x dx$$

- (a) Give an estimate for K using left endpoints with $\Delta x = 1$.
- (b) Give an estimate for K using right endpoints with $\Delta x = 1$.
- (c) Which of the above estimates is closer to K ?
- (d) Give an estimate for K using midpoints with $\Delta x = .1$.
- (e) Give an exact expression for K .

2. Prove, or Disprove and Salvage:

(a) $0 < \int_0^a e^{-x^2} dx < a$

(b) $\int_0^\pi \sin(x^2) dx < 0$

(c) $\int_{-\pi}^\pi e^{-x^2/\sqrt{2}} dx = 2 \int_0^\pi e^{-x^2/\sqrt{2}} dx$

(d) $\int_0^1 \frac{1}{\sqrt{1+x^4}} dx < \int_1^2 \frac{1}{\sqrt{1+x^4}} dx$

(e) $\int_{-3}^3 \frac{x}{1+x^4} dx > 0.001$

(f) The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is greater than $2ab$ and less than $4ab$.

(g) If $f(x)$ is continuous for all x then $\int_a^b f(x+c) dx = \int_{a+c}^{b+c} f(x) dx$