Calculus I, 21-111
Review problems for the second test
March 26

1. Sketch the following curves:
   (a) $y = x^3 - \frac{3}{2}x^2 - 6x$
   (b) $y = x^4 - 4x^3$
2. Determine two numbers such that the product of one and the square of the other is 32, and their sum is as small as possible.
3. The demand curve for a product is given by $p = 50 - .2x$ and the cost of producing x units of the product is $C(x) = .1x^2 + 5x + 96$.
   (a) What quantity of the product will yield the maximum profit?
   (b) What will the selling price be at the quantity producing the maximum profit?
4. Mooncents Coffee sells freshly baked double chocolate cookies at a steady rate all day – a total of 144 a day. The manager wants to know how many cookies to make in each batch. Preparing a batch costs \$8 in employee time and it takes \$.25 worth of shelf space to store one cookie while awaiting sale. How many cookies should they make in each batch to minimize the cost in employee time and storage?
5. A farmer wants to fence a pasture at the intersection of two roads so that it will have a road along an end and a side. He also wants to partition it by a fence down the middle and parallel to one side. The fence next to a road costs \$8 a yard and the rest of the fence costs \$4 a yard. If the farmer has \$960 to spend on the fence what is the largest pasture he can make?
6. Find the point on the line $y = 4x + 5$ closest to the origin.
7. Let the functions f and g satisfy $f(2) = 4$, $f'(2) = 3$, $g(2) = -2$, and $g'(2) = 3$. Determine the value of each of the following:
   (a) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ where $x = 2$.

(b) $\frac{d}{dx}\left(f(x)g(x)\right)$ where $x = 2$.

8. Let the functions f and g satisfy $f(4) = 5$, $f'(4) = -3$, $g(2) = 4$, and $g'(2) = 3$. Determine the value of the following derivative where $x = 2$:

$$\frac{d}{dx}\left[f\left(g(x)\right)\right]$$

9. Given the equation $x^2 - y^2 - xy = 5$, determine the slope of the tangent line to the graph at $(3, 1)$.

10. At noon, a ship is 100 km west of San Francisco. It sails due north at 30 km/hr. How fast is the distance from the ship to the city changing at noon? At 4 pm?