Review Solutions

#1) $x^3 - 6x^2 + 4x = x(x^2 - 6x + 4)$

$x^2 - 6x + 4 = 0$ if $x = \dfrac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$

$$= \frac{6 \pm \sqrt{20}}{2}$$

$$= \frac{6}{2} \pm \frac{2\sqrt{5}}{2}$$

$$= 3 \pm \sqrt{5}$$

So $x^2 - 6x^2 + 4x = 0$ if $x = 0$ or $x = 3 + \sqrt{5}$ or $x = 3 - \sqrt{5}$

#2) $\dfrac{(8x^2y)^{\frac{2}{3}}}{(xy^{-2})^{-1}} = \dfrac{(8x^2y)^{2/3}}{\frac{1}{xy^2}} = (8x^2y)^{2/3}xy^{-2}$

$$= \frac{(8x^2y)^{2/3}x}{y^2}$$

$$= \frac{(64x^4y^2)^{1/3}x}{y^2}$$

$$= \frac{4x^{4/3}y^{2/3}x}{y^2}$$

$$= \frac{4x^{7/3}}{y^{4/3}} \quad \text{or} \quad \left(\frac{64x^7}{y^4}\right)^{1/3}$$

#3) a) $f(g(x)) = \dfrac{\frac{4}{x}}{2+\frac{4}{x}} = \dfrac{4}{x\left(2+\frac{4}{x}\right)} = \dfrac{4}{2x+4}$

b) $f(x)+g(x) \cdot \dfrac{x}{2+x} + \dfrac{4}{x} = \dfrac{x}{2+x} \circ \dfrac{x}{x} + \dfrac{4}{x} \circ \dfrac{(2+x)}{(2+x)}$

$$= \frac{x^2}{(2+x)x} + \frac{8+4x}{x(2+x)}$$

$$= \frac{x^2+4x+8}{x(2+x)}$$

$$= \frac{x^2+4x+8}{x^2+2x}$$

#4) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{(x+h)^2+5(x+h)-(x^2+5x)}{h}$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+\cancel{5x}+5h-\cancel{x^2}-\cancel{5x}}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2+5h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h+5)}{\cancel{h}}$$

$$= \lim_{h\to 0} 2x+h+5 = 2x+5$$

So $f'(x) = 2x+5$

#5) $f(1) = 2\cdot 1^2 + 3\cdot 1 = 5$

So the tangent line passes through the point $(1,5)$

The slope of the tangent line is $f'(1) = 4\cdot 1 + 3 = 7$

So the equation for the tangent line is

$$y - 5 = 7(x-1)$$

Put it in $y = mx+b$ form:

$$y - 5 = 7(x-1)$$
$$y - 5 = 7x - 7$$
$$\boxed{y = 7x - 2}$$

#6) To find the slope of $3x+5y=6$, put it in $y=mx+b$ form:

$$3x + 5y = 6$$
$$5y = -3x + 6$$
$$y = -\frac{3}{5}x + \frac{6}{5} \qquad \text{slope} = -\frac{3}{5}$$

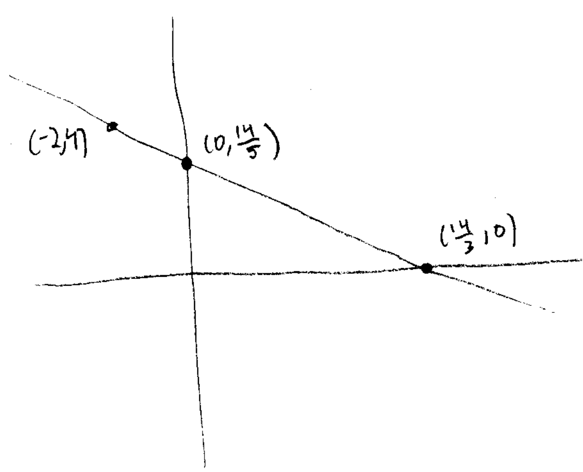
The equation of a line through $(-2,4)$ w/ slope $-3/5$:

$$y - 4 = -\frac{3}{5}(x+2)$$

Put it in $Ax+By=C$ form:

$$y - 4 = -\frac{3}{5}x - \frac{6}{5}$$
$$y = -\frac{3}{5}x + \frac{14}{5}$$
$$\boxed{\frac{3}{5}x + y = \frac{14}{5}}$$

Graph:



#7) a) $s(0) = -16 \cdot 0^2 + 48 \cdot 0 + 64 = 64$ feet

b) $s'(0) = -32 \cdot 0 + 48 = 48$ ft/sec

c) $-16t^2 + 48t + 64 = 0$

$-16(t^2 - 3t - 4) = 0$

$-16(t-4)(t+1) = 0$ if $t = 4$ or -1

By context the answer is when $t = 4$

(can also solve this using the quadratic formula)

d) $s'(4) = -32 \cdot 4 + 48 = -80$ ft/sec or 80 ft/sec downward.

e) $s'(t) = -32t + 48 = 0$ when $t = \frac{48}{32} = \frac{3}{2}$

$s''(\frac{3}{2}) = -32 \Rightarrow$ concave down

$\Rightarrow t = \frac{3}{2}$ is a maximum.

f) $s(\frac{3}{2}) = -16(\frac{3}{2})^2 + 48(\frac{3}{2}) + 64 = 172$ feet

#8) $y = x^3 + 6x^2$

$y' = 3x^2 + 6x = 3x(x+2) = 0$ if $x = 0, -2$

	$x<-2$	$-2<x<0$	$x>0$
$3x$	$-$	$-$	$+$
$(x+2)$	$-$	$+$	$+$
y'	$+$	$-$	$+$
	$\nearrow$	$\searrow$	$\nearrow$

max (at -2), min (at 0)

$y'' = 6x + 6 = 0$ if $x = -1$

	$x<-1$	$x>-1$
y''	$-$	$+$

$y(-1) = (-1)^3 + 6(-1)^2 = 5$

So $(-1, 5)$ is a pt. of inflection.

$y(0) = 0$

$y(-2) = (-2)^3 + 6(-2)^2 = 16$

So $(0,0)$ is a min

$(-2,16)$ is a max

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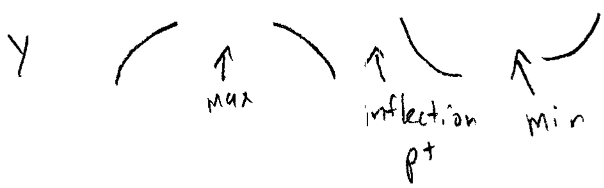
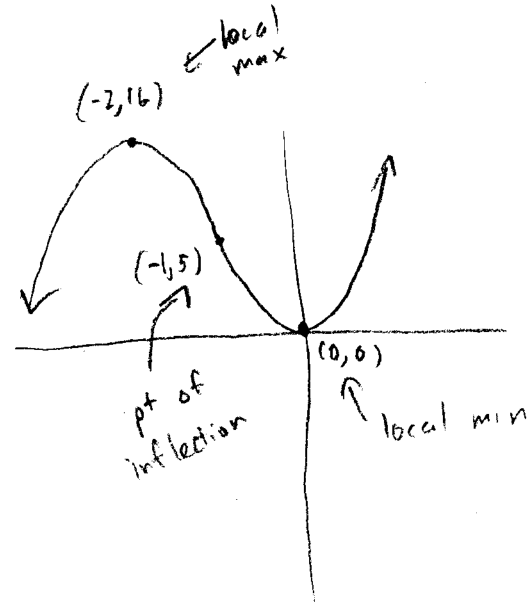
Put it together.

	-2		-1		0	
y'	$+$		$-$	$-$		$+$
y''	$-$		$-$	$+$		$+$





\# 9) Find critical pt: $f'(x) = 3x^2 + 6x = 3x(x+2) = 0$ if $x = 0, -2$

$f(0) = 0^3 + 3\cdot 0^2 + 5 = 5$

$f(-2) = (-2)^3 + 3(-2)^2 + 5 = 9$

So $(0,5)$, $(-2,9)$ are critical pts

		-2		0	
$3x$	$-$		$-$		$+$
$x+2$	$-$		$+$		$+$
f'	$+$		$-$		$+$
		max		min	

So $(0,5)$ is a local min, $(-2,9)$ is a local max.

But since the domain is restricted, we must test endpoints:

We already know $f(-2) = 9$.

$f(2) = 2^3 + 3\cdot 2^2 + 5 = 25$

So $(2,25)$ is the absolute max on the interval, since $25 > 9$.

$(0,5)$ is the absolute min on the interval.