Open Markets in Stochastic Portfolio Theory

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Joint work with Martin Larsson
Stochastic Portfolio Theory

- Introduced by Robert Fernholz (2002)
- **Descriptive** theory of equity markets
- Key feature of SPT models: stability of capital distribution curve
Literature and Shortcomings

• Large body of literature
• Important continuous-time semimartingale models include
  - Atlas Models (Banner, Fernholz, Karatzas (2005); Ichiba et al (2011))
  - Volatility stabilized models (Karatzas, Banner (2005), Pickova (2013))
  - Polynomial models (Filipovic, Larsson (2016), Cuchiero (2019))
• Common model deficiencies related to growth-optimal strategy:
  - Highly-leveraged; egregious short-selling of large-cap stocks,
  - Highly dependent on the size of the market (i.e. number of assets modelled).
Open Markets

• An open market is one where the assets the investor is allowed to invest in change over time.

• This is as opposed to closed markets where the assets are fixed at the initial time.

• Open markets in the context of SPT tend to restrict investment in low-capitalization stocks and only allow investment in high-capitalization stocks
  - Tractability is a concern

• In our open market we have a total of \( d \) assets and allow the investor to invest in
  - \( N \) largest assets \((N \ll d)\)
  - Market portfolio

• Under mild conditions on covariation structure we obtain expressions for growth-optimal strategy in terms of model inputs
Toy Parametric Model

- We introduce a flagship 2 parameter model: **Polynomial Atlas model**

\[
dX_i(t) = \frac{\beta}{2} \left( 1 \{X_i(t) \text{ smallest} \} - X_i(t) \right) dt + \sigma \sum_{j=1}^{d} (\delta_{ij} - X_i(t)) \sqrt{X_j(t)} dW_j(t); \quad \beta, \sigma > 0, \quad i = 1, \ldots, d.
\]

- \( X \) is the market weight process

- Growth Optimal Strategy:

\[
\hat{\pi}_i(t) = \left( 1 - \frac{\beta}{2\sigma^2} \right) X_i(t) 1_{\{X_i(t) \text{ in top } N\}} + \left( 1 + \frac{\beta}{2\sigma^2} \frac{1 - X_0(t)}{X_0(t)} \right) X_i(t) 1_{\{X_i(t) \text{ not in top } N\}}; \quad i = 1, \ldots, d
\]

where \( X_0(t) = \sum_{\{i: X_i(t) \text{ not in top } N\}} X_i(t) \)

- Does not directly depend on \( d \),
- Asymptotic growth-rate converges as \( d \to \infty \),
- Long-only when \( \frac{\beta}{\sigma^2} < 2 \).