Open Markets in Stochastic Portfolio Theory

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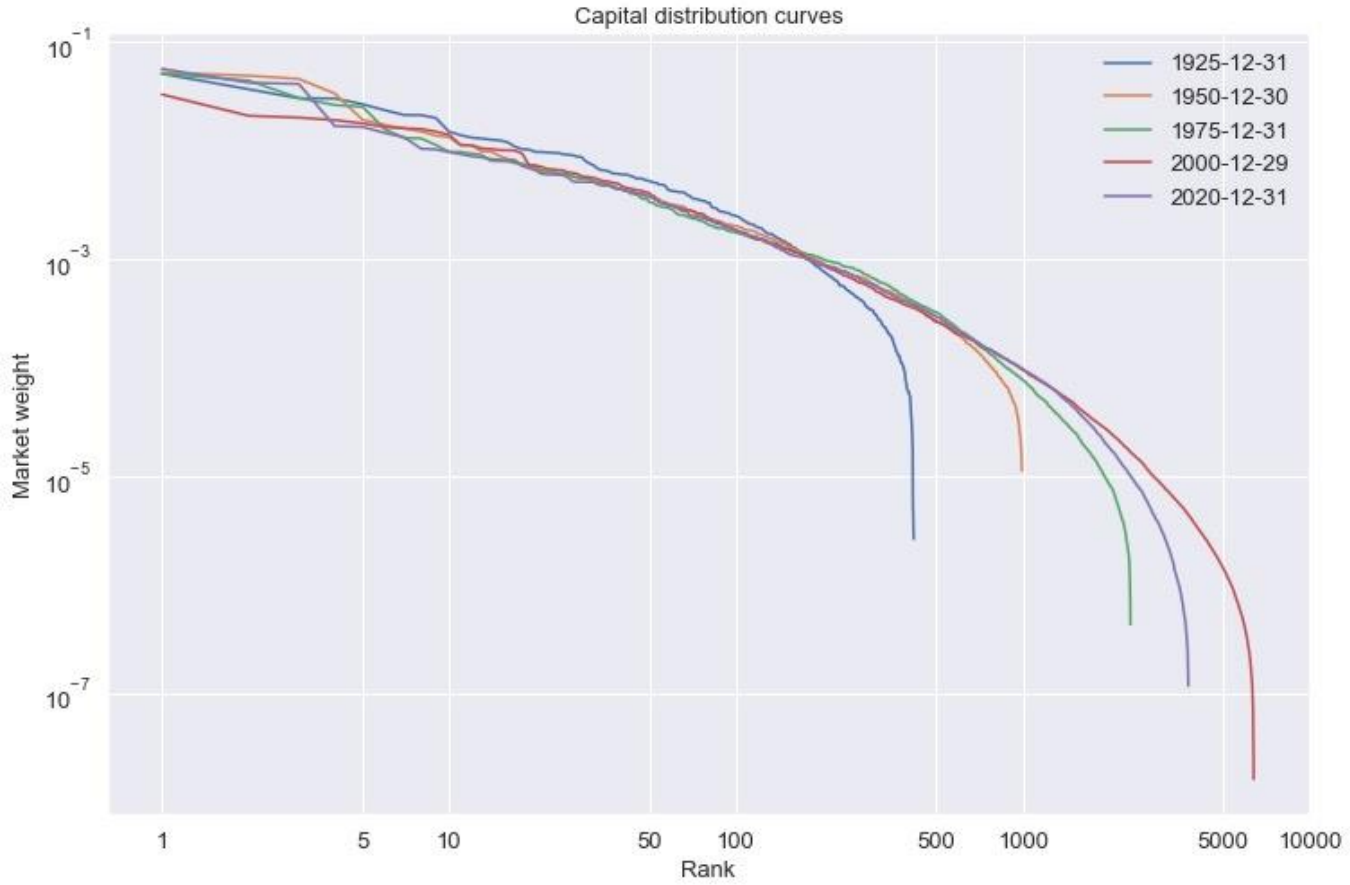
Stochastic Portfolio Theory

- Introduced by Robert Fernholz (2002)
- **Descriptive** theory of equity markets
- Key feature of SPT models: **stability of capital distribution curve**



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Literature and Shortcomings

- Large body of literature
- Important continuous-time semimartingale models include
  - Atlas Models (Banner, Fernholz, Karatzas (2005); Ichiba et al (2011))
  - Volatility stabilized models (Karatzas, Banner (2005), Pickova (2013))
  - Polynomial models (Filipovic, Larsson (2016), Cuchiero (2019))
- Common model deficiencies related to growth-optimal strategy:
  - Highly-leveraged; egregious short-selling of large-cap stocks,
  - Highly dependent on the size of the market (i.e. number of assets modelled).

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Open Markets

- An **open market** is one where the assets the investor is allowed to invest in change over time.
- This is as opposed to *closed markets* where the assets are fixed at the initial time.
- Open markets in the context of SPT tend to restrict investment in low-capitalization stocks and only allow investment in high-capitalization stocks
  - E.g. Fernholz (2018), Karatzas & Kim (2020), Campbell & Wong (2021)
  - Tractability is a concern
- In our open market we have a total of d assets and allow the investor to invest in
  - **N largest assets** $(N \ll d)$
  - **Market portfolio**
- Under mild conditions on covariation structure we obtain **expressions for growth-optimal strategy** in terms of model inputs

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Toy Parametric Model

- We introduce a flagship 2 parameter model: **Polynomial Atlas model**

$$dX_i(t) = \frac{\beta}{2}\left(1_{\{X_i(t)\text{ smallest}\}} - X_i(t)\right)dt + \sigma\sum_{j=1}^{d}(\delta_{ij} - X_i(t))\sqrt{X_j(t)}dW_j(t); \qquad \beta, \sigma > 0, \qquad i = 1, \dots, d.$$

- X is the market weight process

- Growth Optimal Strategy:

$$\hat{\pi}_i(t) = \left(1 - \frac{\beta}{2\sigma^2}\right)X_i(t)1_{\{X_i(t)\text{ in top }N\}} + \left(1 + \frac{\beta}{2\sigma^2}\frac{1 - X_0(t)}{X_0(t)}\right)X_i(t)1_{\{X_i(t)\text{ not in top }N\}}; \qquad i = 1, \dots, d$$

where $X_0(t) = \sum_{\{i:X_i(t)\text{ not in top }N\}} X_i(t)$

- Does not directly depend on d,
- Asymptotic growth-rate converges as $d \to \infty$,
- Long-only when $\frac{\beta}{\sigma^2} < 2$.

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