Math 300 Class 21

Wednesday 27th February 2019

Definition 1 — *Countably infinite, countable and uncountable sets*
A set X is **countably infinite** if there is a bijection $\mathbb{N} \to X$. A set is **countable** if it is finite or countably infinite, and is **uncountable** if it is not countable.

Exercise 2
Prove that $\mathscr{P}(\mathbb{N})$ is uncountable.

Theorem 3 — *Some facts about countability*

(i) If there is an injection $X \to \mathbb{N}$, then X is countable.

(ii) If there is a surjection $\mathbb{N} \to X$, then X is countable.

(iii) Properties (i) and (ii) remain true if $\mathbb{N}$ is replaced by any countably infinite set.

(iv) If X and Y are countable, then $X \times Y$ is countable.

(v) The union of countably many countable sets is countable.

Example 4

Prove that $\mathbb{Q}$ is countable by defining a surjection from a countable set to $\mathbb{Q}$.

Example 5

Prove that $\mathbb{Q}$ is countable by expressing it as a union of countably many countable sets.

Example 6

Prove that $\binom{\mathbb{N}}{2}$, the set of all subsets of $\mathbb{N}$ of size 2, is countably infinite.