Math 300 Class 11

Friday 1st February 2019

Definition 1
Let $f : X \to Y$ be a function and let $U \subseteq X$. The **image of** U **under** f is the subset $f[U] \subseteq Y$ defined by

$$f[U] = \{f(x) \mid x \in U\} = \{y \in Y \mid \exists x \in U, y = f(x)\}$$

That is, $f[U]$ is the set of values that the function f takes when given inputs from U.

The '**image of** f' is the image of its entire domain, i.e. the set $f[X]$.

Example 2
Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2$ for all $x \in \mathbb{R}$. Find $f[\mathbb{R}]$ and $f[(-1,1)]$.

Example 3
Define $g : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by $g(a,b) = \dfrac{a}{1+|b|}$ for all $(a,b) \in \mathbb{R} \times \mathbb{R}$.

Find a subset $V \subseteq \mathbb{R} \times \mathbb{R}$ such that $g[V] = \mathbb{Q}$.

Example 4

Let $f : X \to Y$ be a function and let $U, V \subseteq X$.

Prove that $f[U \cap V] \subseteq f[U] \cap f[V]$.

Give an example to show that $f[U] \cap f[V]$ need not be a subset of $f[U \cap V]$.

Definition 5

Let $f : X \to Y$ be a function and let $V \subseteq Y$. The **preimage of V under f** is the subset $f^{-1}[V] \subseteq X$ defined by

$$f^{-1}[V] = \{x \in X \mid f(x) \in V\} = \{x \in X \mid \exists y \in V, y = f(x)\}$$

Example 6

Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2$ for all $x \in \mathbb{R}$. Find $f^{-1}[\mathbb{R}]$ and $f^{-1}[(-\infty, 4)]$.

Example 7

Let $f : X \to Y$ be a function. Prove that $f^{-1}[U \cap V] = f^{-1}[U] \cap f^{-1}[V]$ for all subsets $U, V \subseteq Y$.

Example 8

Let $f : X \to Y$ and $g : Y \to Z$ be functions, and let $W \subseteq Z$. Prove that $(g \circ f)^{-1}[W] = f^{-1}[g^{-1}[W]]$.