Math 300 Class 1

Monday 7th January 2019

Definition 1 — *Propositions and proofs*
A **proposition** is a statement to which it is possible to assign a **truth value** ('true' or 'false'). If a proposition is true, a **proof** of the proposition is a logically valid argument demonstrating that it is true, which is pitched at such a level that a member of the intended audience can verify its correctness.

Exercise 2
Think of an examples of (a) a true proposition, (b) a false proposition, (c) a proposition whose truth value you don't know, and (d) a statement that is not a proposition.

(a)

(b)

(c)

(d)

One of the most fundamental concepts in all of pure mathematics is that of a *set*.

Definition 3 — *Sets and elements (informal definition)*
A **set** is a collection of objects. The objects in the set are called **elements** of the set. If X is a set and x is an object, then we write $x \in X$ to denote the assertion that x is an element of X.

We will study sets abstractly soon. For now, we consider the following concrete examples:

- The set $\mathbb{N}$ of **natural numbers**: $0, 1, 2, 3, \ldots$
- The set $\mathbb{Z}$ of **integers**: $\ldots, -2, -1, 0, 1, 2, \ldots$
- The set $\mathbb{Q}$ of **rational numbers**, which are numbers of the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}$ and $b \neq 0$;
- The set $\mathbb{R}$ of **real numbers**, which are arbitrary points on the 'number line'.

So for example, '$5 \in \mathbb{Z}$' means '5 is an integer', and '$\frac{1}{2} \in \mathbb{Q}$' means '$\frac{1}{2}$ is a rational number'.

Definition 4 — *Division of integers*
An integer b **divides** an integer a if there is an integer q such that $a = qb$.

For example, 4 divides 12, since $3 \times 4 = 12$.

Exercise 5
Given any integer a, prove that 1 divides a and that a divides 0.

Proposition 6
Let a, b and c be integers. If c divides b and b divides a, then c divides a.

Proof

□

Theorem 7 — *Division theorem*
Let $a, b \in \mathbb{Z}$ with $b \neq 0$. There is exactly one way to write

$$a = qb + r$$

such that q and r are integers, and $0 \leqslant r < |b|$.

The integer q is called the **quotient** and the integer r is called the **remainder**.

Proposition 8
Let $a, b \in \mathbb{Z}$ with $b > 0$, and suppose that a leaves a remainder $r > 0$ when divided by b. Then $-a$ leaves a remainder of $b - r$ when divided by b.

Proof

$\square$

Pre-class assignment for Class 2 (Tue, Jan 8)

Read the course syllabus carefully and then fill out the following questionnaire:

https://goo.gl/forms/U4qtK3yhDXNWiP3m2

The syllabus and questionnaire can both be found on Canvas (go to Assignments → Class 2).