Foundations of Higher Mathematics (Math 300)

Section 31, Fall 2018

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Section 1
Times and places

- **Class** — MoWeFr 9:00–9:50am in Technological Institute MG51

  The focus of class will be on learning material and acquiring skills (see the learning objectives below). Classes will include a mixture of lecture and activities, including problem-solving exercises, group discussions and student-led presentations.

- **Discussion** — Tu 9:00–9:50am in Technological Institute MG51

  Discussion sessions will be led by Mingyi. They will be less formally structured than classes, and are intended to be an opportunity for you to review the material covered in class and to strengthen your learning.

- **Office hours.** These are times when I and/or Mingyi will be available to discuss pretty much anything from the course—they are a great opportunity for us to get to know you better (and vice versa). Please see the guidelines for office hours in Section 6 below.

  Office hours will be announced during class.

  If you can't attend the scheduled office hours, please contact me to schedule a meeting at another time.

- **Examinations.** There will be a midterm exam, which will be held during class, and a final exam—you might want to put the following dates in your calendar!
  - ◇ **Midterm exam.** Wednesday 31st October during class.
  - ◇ **Final exam.** Thursday 13th December at 3:00–5:00pm — location TBA.

Section 2

Learning objectives

Upon successful completion of this course you should be able to:

(1) Accurately use standard mathematical notation and terminology in mathematical writing, including logical operators, quantifiers, sets and set operations, functions, equivalence relations, binomial coefficients and factorials;

(2) Write correct, clear and precise mathematical proofs, in an appropriate level of detail, of both familiar and unfamiliar mathematical results in the areas covered;

(3) Recognise and apply standard proof techniques, including direct proof, contradiction, contraposition, and weak and strong mathematical induction;

(4) Accurately recall mathematical definitions and state and prove the main theorems in the mathematical areas covered;

(5) Solve standard unseen problems in the mathematical areas covered by identifying which proof techniques and results from the course are appropriate and applying them accurately;

(6) Typeset basic mathematical documents using LaTeX, including the mathematical notation used in the course, sections, references and labels, and basic formatting.

Descriptions of some of the topics we will cover are as follows.

- **Fundamentals.** In the first few weeks we will set the scene for the rest of the course. We will learn some elementary proof techniques (including proof by contradiction and proof by induction); then we will develop a system of symbolic logic, and study the absolutely fundamental mathematical notions of *sets* and *functions*.
- **Finite and infinite sets.** We will use functions to pin down a precise notion of *size* that allows us to count the elements in a finite set. After developing some mathematical tools for counting, we will generalise this notion to study and compare the sizes of *infinite* sets.
- **Relations.** A fundamental notion in mathematics is that of a *relation*, and particularly that of an *equivalence relation*. We will study different kinds of relations, and prove that equivalence relations are 'the same thing' as *partitions*.
- **Additional topic(s).** If time permits, I will poll the class to choose a topic to cover next. Some possible options include elementary number theory, real analysis, order theory and structural induction.
- **LaTeX.** Typesetting mathematical documents is very difficult in most Office-style 'what you see is what you get' (WYSIWYG) editing software. The *de facto* standard for typesetting mathematics is LaTeX (pronounced 'lay-tek' or 'lah-tek'), in which all formatting and mathematical notation is entered as code. Many classes will include LaTeX workshops, and the course includes a LaTeX typesetting project.

Section 3

Reading materials

As course notes we will use my (freely available) book-in-progress:

- *An infinite descent into pure mathematics*
  https://infinitedescent.xyz/

We will additionally draw from the following books, which you need not purchase:

- *Book of Proof* by Richard Hammack
  ISBN 978-0989472104
  https://www.people.vcu.edu/~rhammack/BookOfProof/
- *Reading, Writing, and Proving* (2nd edition) by Ulrich Daepp & Pamela Gorkin
  ISBN 978-1441994783

The following books provide useful supplemental reading:

- *A Concise Introduction to Pure Mathematics* (4th edition) by Martin Liebeck
  ISBN 978-1498722926
- *How to Prove It: A Structured Approach* (2nd edition) by Daniel J. Velleman
  ISBN 978-0521675994
- *Invitation to discrete mathematics* by Jiří Matoušek and Jaroslav Nešestřil
  ISBN 978-0198570424

A fun, non-technical discussion about how the skills you acquire when studying abstract mathematics relate to the real world is:

- *How Not to Be Wrong: The Power of Mathematical Thinking* by Jordan Ellenberg
  ISBN 978-0143127536

Section 4

Assessment and grades

Below are descriptions of the course assessments and grade assignments.

- **Classwork** (10% of total grade)**.**
  - ◇ **What?** Various activities and problem-solving exercises in class, often coupled with a small amount of work (e.g. reading or watching a video) to be done before class.
  - ◇ **Why?** To keep you engaged with the material as it is presented and to give you the opportunity to make errors in a low-stakes environment.
- **Homework** (25% of total grade)**.**
  - ◇ **What?** Several challenging mathematical exercises based on material covered in class, graded for mathematical correctness and quality of proof-writing.
  - ◇ **Why?** To give you the opportunity to show off what you've learnt, and to allow me to give you individual feedback on your progress.
- **Midterm examination** (20% of total grade)**.**
  - ◇ **What?** A fifty minute exam during class time on Wednesday 31st October.
  - ◇ **Why?** To solidify the knowledge and skills you acquired during the first half of the quarter, and to serve as a milestone in the course.
- **Final examination** (35% of total grade)**.**
  - ◇ **What?** A two hour exam from 3:00–5:00pm on Thursday 13th December.
  - ◇ **Why?** To test you on the knowledge and skills you have acquired throughout the quarter.
- **LaTeX project** (10% of total grade)**.**
  - ◇ **What?** Short write-up of a document using LaTeX, putting into practice the mathematical typesetting skills you'll learn throughout the course.
  - ◇ **Why?** To familiarise you with using LaTeX to typeset mathematical (and non-mathematical) documents.

The preliminary grade borderlines are as follows:

		B+	87%	**C+**	77%		
A	93%	**B**	83%	**C**	73%	**D**	60%
A-	90%	**B-**	80%	**C-**	70%		

These borderlines might be lowered, but will not be raised; for instance, a score of 83% guarantees you a grade of B or higher, even if the borderlines change.

Section 5

Tentative class schedule

Date			Sec.	Topic	Notes
Week 1	Friday	28th September	1.1	Welcome, getting started	
Week 2	Monday	1st October	1.2	Proof techniques	
	Tuesday(!)	2nd October	1.2	Proof techniques	
	Wednesday	3rd October	2.1	Symbolic logic	Hw 1 due
	Friday	5th October	2.1	Symbolic logic	
Week 3	Monday	8th October	2.1	Symbolic logic	
	Wednesday	10th October	1.3	Induction on $\mathbb{N}$	Hw 2 due
	Friday	12th October	1.3	Induction on $\mathbb{N}$	
Week 4	Monday	15th October	1.3	Induction on $\mathbb{N}$	
	Wednesday	17th October	2.2	Sets and set operations	Hw 3 due
	Friday	19th October	2.2	Sets and set operations	
Week 5	Monday	22nd October	2.3	Functions	
	Wednesday	24th October	2.3	Functions	Hw 4 due
	Friday	26th October	4.1	Functions	
Week 6	Monday	29th October	4.1	Functions	
	Tuesday(!)	30th October	—	*Midterm review*	
	Wednesday	31st October	—	**Midterm examination**	
	Friday	2nd November	—	*No class*	
Week 7	Monday	5th November	4.2	Counting principles	
	Tuesday(!)	6th November	4.2	Counting principles	
	Wednesday	7th November	4.3	Infinite sets	
	Friday	9th November	4.3	Infinite sets	
Week 8	Monday	12th November	7.1	Discrete probability spaces	
	Wednesday	14th November	7.1	Discrete probability spaces	Hw 5 due
	Friday	16th November	7.2	Discrete random variables	
Week 9	Monday	19th November	7.3	Expectation	
	Wednesday	21st November	—	*No class*	Hw 6 due
	Friday	23rd November	—	*No class*	
Week 10	Monday	26th November	5.1	Relations	
	Wednesday	28th November	5.1	Equivalence relations	
	Friday	30th November	5.2	Order relations	Hw 7 due†
Reading period	Monday	3rd December	—	*Final review*	Optional
	Wednesday	5th December	—	*Final review*	Optional
	Friday	7th December	—	*Final review*	Optional
	Thursday	13th December	—	**Final examination 3:00–5:00pm**	

†: Hw 7 has a 'soft deadline' and will also be accepted during the reading period

Section 6

Policies and guidelines

Academic honesty and integrity

My stance on academic honesty is simple: **all work you submit should accurately reflect your understanding**, and **any help you receive should be acknowledged**. This means that if someone were to ask you to explain your work, then you would be able to explain it and to say how you came to know it. What follows are some more specific descriptions of what this looks like in practice.

Collaboration. Speaking to each other about the course material and homework problems is one of the most effective ways to learn, so this is encouraged. What I ask is that you:

- **Cite your collaborators.** This means that you're giving them credit for their help, and avoids plagiarism issues. Just write a sentence at the end of your work saying who you worked with, e.g. 'I discussed Q4 with Carl Gauss, who helped me prove that f is surjective'.
- **Write your work up independently.** If you made any permanent records (such as notes or photos) during collaboration sessions, these records should be destroyed well before you write up your solution. For example, any notes on whiteboards should be erased, notes on paper should be thrown away, and photos should be deleted. Direct copying is forbidden.

External resources. Sometimes you need a little more guidance than is available from your notes, and doing some research can give you the boost you need to understand the material in the course. If you do use external resources, then please:

- **Cite your sources.** If you used a book or website, other than the course textbook or other assigned reading, please say so—just the book title and author, or web page URL, is fine.
- **Write your work up independently.** Close the book or web page and make sure you've understood what you learnt before you start writing—otherwise, all you're really doing is copying, and then your work doesn't reflect your understanding.

Examinations. Exams are the main so-called *summative assessments* of the course, meaning that they are intended to be an opportunity for you to demonstrate the knowledge and skills you have acquired. The only resource you should have available to you going into an exam is your brain—this means you should not have your notes open, you should not speak to others during the exam, and you should not be looking at other people's answers.

Instances of suspected academic integrity violations will be reported to the Weinberg Assistant Dean for Academic Integrity for further investigation. If I believe academic integrity violations are widespread, then I may resort to using plagiarism detection software.

If you are ever in doubt about whether something you are doing is in violation of academic integrity, your safest bet is to ask me as soon in advance of turning in your work as possible.

Attendance

Classes will be both fun and intellectually stimulating, so you'll probably want to attend, but in case that's not enough motivation, remember that 10% of your grade comes from classwork. If you know you're going to miss a class, **please let me know as soon as possible** so that I can tell you what to do in order to catch up.

Frequent absence from class is correlated with lower academic performance, as well as other issues including mental health concerns—with this in mind, if I notice that you have developed a pattern of absence, then I will contact the Weinberg Office of Undergraduate Studies and Advising (OUSA), who will then check in with you.

If you are absent from an examination, I will work with you and your academic advisor (and/or OUSA) to work out if, how and when you can make up the work. Please let me know during the first two weeks of the quarter if you know in advance that you must miss an examination for a legitimate reason.

Homework submission

Homework is designed not just to test your knowledge, but also to help you learn. I set deadlines because it's important that you understand the content on the homework before we move on to new material in class. As such, it is in your own interests to do the homework and submit it on time.

Late homework will typically not receive credit since the grading turnaround will be quick. If there are any special circumstances that mean you absolutely need to submit the homework late, please speak to me as soon as possible, and preferably well in advance of the deadline. If in doubt, ask—the sooner you ask, the easier it will be for us to find a solution.

Contesting a grading error

Mingyi and I will grade a *lot* of your work throughout the quarter, but (just like you) we are mere humans, so it is entirely possible that we will make an error from time to time. If you believe a grading error has been made, then please do the following:

1. Write a note indicating what error(s) you believe has been made—be as specific as possible, and include the relevant question number(s).
2. Attach the note to your work and hand it in at the beginning of class within three working days of the return date—for example, if homework was returned on a Friday, you should turn it in on the following Monday or Wednesday.

I will then regrade the work in question from scratch and return it to you within a week—at this point, your homework grade is final.

Extra credit

If you feel particularly enthusiastic about a topic, please speak to me—we might be able to organise some work that you can do for extra credit, such as giving a short presentation in class, writing a blog post or making a poster.

Extra credit will be reflected in your grade by increasing your overall course percentage by an agreed-upon fixed amount (usually 0.5% or 1%), up to a maximum total of 1.5% for the quarter.

Extra credit is not guaranteed. I will make every effort to provide as many opportunities as I can, but this is limited by my own resources—in particular, if you are interested, then you should ask as early in the quarter as you can. All work for extra credit must be completed before the Weinberg College Reading Period.

Make-up work

One of the best ways to learn is by making errors and then reflecting on those made. With this in mind, I will provide the opportunity to make up one question from each homework assignment by resubmitting your solution together with a written reflection exercise.

Although you will not be able to resubmit your midterm or final exam solutions for make-up credit, reflecting on your midterm performance will be part of subsequent classwork and homework.

Classwork cannot be made up. To put this in perspective, each class is worth approximately 0.4% of your total grade, so the effect of missing one or two classes (due to sickness, for example) is negligible—however, missing several classes will have a noticeable negative impact on your grade.

Accommodations

It is very important to me that my methods of instruction and assessment are fair to everyone. If you require any accommodations, including extra time on examinations, note-taking services, large-text format materials, alternative test locations, then you should register with AccessibleNU:

- **AccessibleNU**

  https://www.northwestern.edu/accessiblenu/

  (847) 467-5530 / accessiblenu@northwestern.edu

I am only able to make accommodations after I have received an accommodation notification from AccessibleNU—I cannot accept any other forms of evidence of need for accommodations, such as notes from doctors or emails from parents.

Mental health and wellness

It is likely that you will feel overwhelmed (with work or otherwise) at some point during the quarter. Please be aware that Northwestern University has a *lot* of resources available for helping you on your way, including:

- **Counseling and Psychological Services (CAPS)** (confidential resource)
  https://www.northwestern.edu/counseling/
  (847) 491-2151
- **Student Assistance and Support Services (SASS)**
  https://www.northwestern.edu/studentaffairs/dos/
  (847) 491-4582
- **Dean on Call** (if you don't know who else to call)
  (847) 491-8430 (Mo–Fr 8:30am–5:00pm)
  (847) 467-3022 (after hours)

More information and resources can be found on the university's Health and Wellness page:

https://www.northwestern.edu/studentaffairs/dos/resources/health-wellness/

Discrimination, harassment and sexual misconduct

We share a responsibility to make our community one where everyone has equal opportunities and feels safe, both inside and outside the classroom. I believe discrimination and harassment (sexual or otherwise) have no place in my classes or at Northwestern University, and I am committed to preventing such behaviour and taking action if it occurs. Some resources for students regarding discrimination and harrassment are as follows.

- **Center for Awareness, Response and Education (CARE)** (confidential resource)
  https://www.northwestern.edu/care/
  (847) 491-2054
- **Deputy Title IX Coordinator for Students**
  (847) 467-6571 / deputytitleixcoordinator@northwestern.edu
- **Office of Equity**
  https://www.northwestern.edu/equity/
  (847) 467-6165

Whilst I am willing to discuss sensitive issues, please bear in mind that I am required to report instances of suspected sexual misconduct to the Deputy Title IX Coordinator for Students.

Office hours

Office hours are quite simply times that I and Mingyi have set aside to be available to meet with students. Make the most of them! Since office hours are entirely unstructured, please make sure that you follow the following guidelines.

- Have something in mind to discuss or ask, and be as specific as possible.
- Please take turns with other students—for example, if you ask a question, allow the other students present to ask a question before you ask a follow-up question.
- If office hours are busy, please avoid 'camping out'—my office isn't very big!
- Please don't show me your written attempts at homework solutions. (I won't look at them!) It is not fair for me to verify if your attempt is correct or to suggest how it can be improved ahead of the submission deadline.
- Please only come to the office hours for this course or 'open office hours'—if you come to my Math 290 office hours, I will prioritise questions about Math 290 (and vice versa for Math 290 students attending Math 300 office hours).

Talk to me

I want you to learn a lot and I want you to enjoy taking this course. So that I can find out if this is happening, I encourage feedback—be it positive or negative—on all aspects of the course at any time during the quarter.

For example, if something I'm doing is making it difficult for you to learn, then say something before it's too late; or if you particularly enjoyed something we did in class, say so so that we can do it again.

You can do this by just speaking to me or Mingyi, by sending one of us an email, or by using the anonymous comments form, details of which will be circulated in class. Please bear in mind that I cannot reply individually to anonymous feedback. Giving feedback will in no way affect your grade, positively or negatively.