Math 290-2 Class 11

Monday 4th February 2019

Coordinate systems

The **cartesian coordinates** (named after René Descartes) of a point P in $\mathbb{R}^n$ tell us how many units must be traversed from the origin in the direction of each standard basis vector to get to P. But this is not the only way of specifying a point—it may be more convenient to use a different coordinate system, such as one of those described below.

The **polar coordinates** of a point P in $\mathbb{R}^2$ are given by (r, θ), where:

- r is the distance of P from the origin; and
- θ is the angle of the ray from the origin on which P lies.

We may allow $r < 0$, in which case P lies $|r|$ units on the ray with angle $\theta + \pi$.

The conversion between cartesian and polar coordinates is given by

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \text{and} \quad \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \dfrac{y}{x} \text{ or indeterminate if } x = 0 \end{cases}$$

The **cylindrical coordinates** of a point P in $\mathbb{R}^3$ are given by (r, θ, z), where:

- (r, θ) are the polar coordinates of the projection of P onto the (x, y)-plane; and
- z is the height of P along the z-axis.

The conversion between cartesian and cylindrical coordinates is given by

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \quad \text{and} \quad \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \dfrac{y}{x} \\ z = z \end{cases}$$

The **spherical coordinates** of a point P in $\mathbb{R}^3$ are given by (ρ, φ, θ), where:

- ρ is the distance of P from the origin;
- φ is the angle that $\overrightarrow{OP}$ makes with the positive z-axis; and
- θ is the angle of P about the z-axis.

The conversion between cartesian and spherical coordinates is given by

$$\begin{cases} x = \rho\cos\theta\sin\varphi \\ y = \rho\sin\theta\sin\varphi \\ z = \rho\cos\varphi \end{cases} \quad \text{and} \quad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\varphi = \dfrac{\sqrt{x^2+y^2}}{z} \\ \tan\theta = \dfrac{y}{x} \end{cases}$$

1. Find the equation of the circle $(x-3)^2+y^2=9$ in polar coordinates.

2. Sketch the curve in $\mathbb{R}^2$ described in polar coordinates by the equation $r=\theta$ (for $r \geqslant 0$).

3. (a) Sketch the surface in $\mathbb{R}^3$ whose equation in cylindrical coordinates is $z = 2r$.

(b) Find the cartesian coordinates of the surface you just sketched.

4. Sketch the solid region of $\mathbb{R}^3$ described in spherical coordinates by

$$0 \leqslant \rho \leqslant 1, \quad 0 \leqslant \varphi \leqslant \frac{\pi}{4}, \quad 0 \leqslant \theta \leqslant \pi$$

5. Find a way of converting between cylindrical and spherical coordinates.