1. Find the vector equation of the line $2x+3y=1$ in $\mathbb{R}^2$.

Point on line : $(-1, 1)$

Direction of line : $(-3, 2)$

$\Rightarrow$ The vector eq^{n} is $\vec{r}(t) = (-1, 1) + t\,(-3, 2)$.

2. Find the vector equation of the line of intersection of the planes in $\mathbb{R}^3$ given by

$$x+y-z=-1 \quad \text{and} \quad x+2y-2z=1$$

Solving the system:

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 1 & 2 & -2 & 1 \end{array}\right) \xrightarrow{R_2-R_1} \left(\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 2 \end{array}\right) \xrightarrow{R_1-R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -1 & 2 \end{array}\right)$$

$x = -3, \quad z = t \text{ (free)}, \quad y = t + 2$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 + 0t \\ 2 + t \\ 0 + t \end{pmatrix}$$

So the vector equation is

$$\vec{r}(t) = (-3, 2, 0) + t\,(0, 1, 1)$$

3. Show that every point (x,y,z) on the line in $\mathbb{R}^3$ with vector equation $\mathbf{r}(t) = \mathbf{b} + t\mathbf{a}$ satisfies

$$\frac{x-b_1}{a_1} = \frac{x-b_2}{a_2} = \frac{x-b_3}{a_3}$$

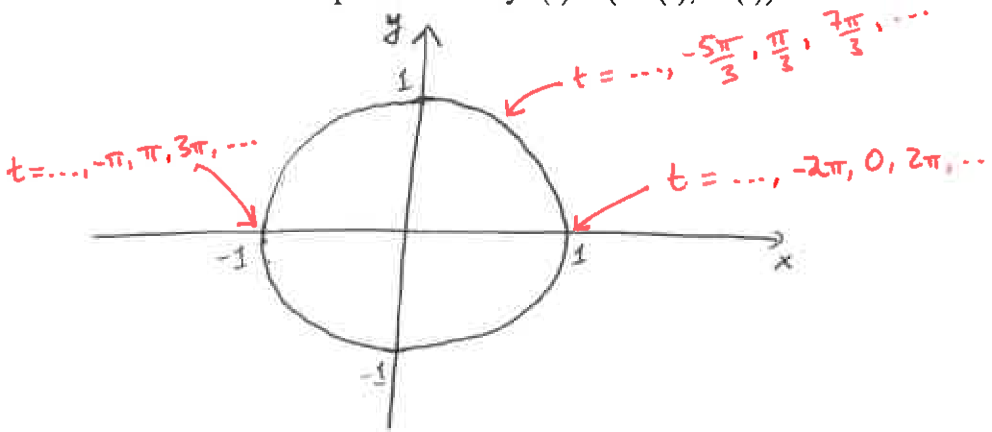
as long as $a_1, a_2, a_3 \neq 0$. This is called the **symmetric form** of the line.

$$(x, y, z) = (b_1 + ta_1,\ b_2 + ta_2,\ b_3 + ta_3)$$

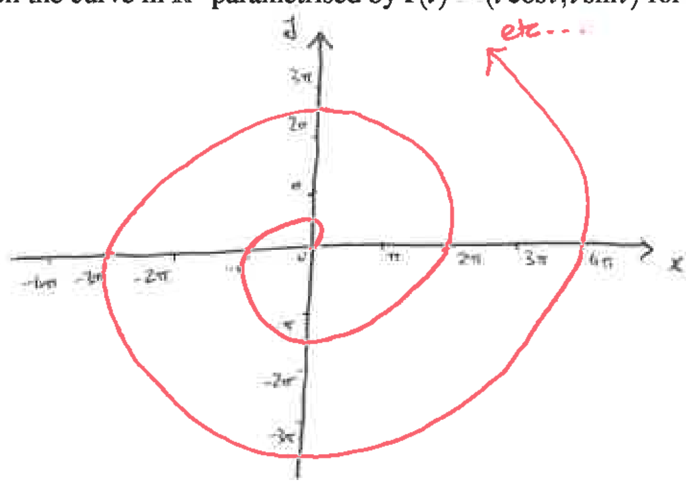
Solving for t in each coordinate gives:

$$t = \frac{x-b_1}{a_1} = \frac{y-b_2}{a_2} = \frac{z-b_3}{a_3}$$

4. Sketch the curve in $\mathbb{R}^2$ parametrised by $\mathbf{r}(t) = (\cos(t), \sin(t))$.



5. Sketch the curve in $\mathbb{R}^2$ parametrised by $\mathbf{r}(t) = (t\cos t, t\sin t)$ for $t \geqslant 0$.



6. Sketch the curve in $\mathbb{R}^3$ parametrised by $\mathbf{r}(t) = (\cos t, \sin t, t)$.