Math 290-1 Class 1

Friday 28th September 2018

Linear systems

When solving a system of simultaneous equations, you can…

- Multiply every term in an equation by a nonzero number;

$$\begin{cases} x + 2y = 1 \\ 2x + \ \ y = 0 \end{cases} \xrightarrow{2\times(\mathrm{I})} \begin{cases} 2x + 4y = 2 \\ 2x + \ \ y = 0 \end{cases}$$

- Add any multiple of one row to another row;

$$\begin{cases} x + 2y = 1 \\ 2x + \ \ y = 0 \end{cases} \xrightarrow{(\mathrm{I}) \mapsto (\mathrm{I})-2\times(\mathrm{II})} \begin{cases} -3x \qquad = 1 \\ \ \ 2x + y = 0 \end{cases}$$

- Swap any two rows.

$$\begin{cases} x + 2y = 1 \\ 2x + \ \ y = 0 \end{cases} \xrightarrow{(\mathrm{I}) \leftrightarrow (\mathrm{II})} \begin{cases} 2x + \ \ y = 0 \\ x + 2y = 1 \end{cases}$$

A solution to a system of simultaneous equations corresponds to a common point of intersection of all of the lines or planes (or hyperplanes) described by the equations in system.

A system of 2 equations in 2 variables (x,y) describes two lines, which might…

- Have different slopes $\rightsquigarrow$ unique solution at the point of intersection;
- Be parallel $\rightsquigarrow$ no solutions exist; or
- Be the same line $\rightsquigarrow$ infinitely many solutions, one for each point on the line.

A system of 3 equations in 3 variables (x,y,z) describes three planes, which might…

- Meet at a point $\rightsquigarrow$ unique solution at this point of intersection;
- Meet at a line $\rightsquigarrow$ infinitely many solutions, one for each point on the plane;
- Meet at lines *pairwise* but not all together $\rightsquigarrow$ no solutions;
- Be parallel $\rightsquigarrow$ no solutions; or
- Be the same plane $\rightsquigarrow$ infinitely many solutions, one for each point on the plane.

If the planes meet at a line, the solution can be parametrised by a single variable. (Typical trick that usually [but not always] works: set $z = t$, then use expression for y in terms of z to put y in terms of t, and then use expression for x in terms of y and z to put x in terms of t.)

Examples

1. Just by looking at the following linear systems of 2 equations in 2 variables, indicate next to each system whether you expect it to have no solution, a unique solution, or infinitely many solutions..

$$\begin{cases} x + 2y = 1 \\ 2x + 3y = 1 \end{cases}$$

Number of solutions: none one infinitely many

Explanation:

$$\begin{cases} x + 2y = 1 \\ 2x + 4y = 1 \end{cases}$$

Number of solutions: none one infinitely many

Explanation:

$$\begin{cases} x + 2y = 1 \\ 2x + 4y = 2 \end{cases}$$

Number of solutions: none one infinitely many

Explanation:

$$\begin{cases} 3x - 2y = 4 \\ 2x - y = 1 \end{cases}$$

Number of solutions: none one infinitely many

Explanation:

$$\begin{cases} x - 2y = -1 \\ 2y - x = 0 \end{cases}$$

Number of solutions: none one infinitely many

Explanation:

$$\begin{cases} -6x + 2y = 4 \\ 3x - y = -4 \end{cases}$$

Number of solutions: none one infinitely many

Explanation:

2. [Bretscher §1.1 Q9] Find all the solutions to the following linear system and give a geometric account for your answer.

$$\begin{cases} x + 2y + 3z = 1 \\ 3x + 2y + z = 1 \\ 7z + 2y - 3z = 1 \end{cases}$$

3. [Bretscher §1.1 Q18] Let a, b and c be given. Find all the solutions to the following linear system in terms of a, b and c.

$$\begin{cases} x + 2y + 3z = a \\ x + 3y + 8z = b \\ x + 2y + 2z = c \end{cases}$$

4. [Bretscher §1.1 Q21] The sums of any two of three real numbers are 24, 28 and 30. Find these three numbers.