Multivariate Analysis (21-256)

Clive Newstead, Summer I 2014

Class info
Time: Every weekday at 10:30am–11:50am
Location: Wean Hall 4623
Units: 9 units

Instructor info
Name: Clive Newstead
Office: Wean Hall 8205
Email: cnewstead@cmu.edu

Website: http://math.cmu.edu/∼cnewstea/teaching/21-256/

1 Course outline

Welcome to 21-256! This is a six-week summer course that will teach you how to use a wide range of mathematical tools from multivariate analysis. A student who completes this course with a good grade can expect to feel comfortable using the techniques they'll need to embark upon quantitative study of phenomena in which several measurable variables are at play (e.g. in economics, business, statistics, social sciences). This course is advertised as being intended for economics and business majors, but in reality is suitable for many more.

What and why

Multivariate Analysis is split into three parts:

(1) **Vectors and matrices.** Many real-world problems can be modelled *linearly*, roughly meaning that the variable quantities in the problem are related to each other by constant multiples. Even when this isn't the case, a process called *linearization* makes it possible to translate more complicated problems into this relatively simple setting to a high degree of accuracy. The first part of the course you will learn how to use vectors and matrices to model and solve linear problems.

(2) **Optimization.** If you run a business, you probably want to maximize your profits and minimize your costs, and yet there are a certain number of resources you require. This is a typical example of an *optimization problem*. Such problems appear in industry and throughout the social and physical sciences when you need to maximize a quantity subject to some constraints. In the second part of the course, you will learn how to tackle this kind of problem by combining calculus with what you learnt about vectors.

(3) **Integration.** The third section on the course will cover integration of multivariate functions, which is essential in applications to data analysis and probability theory: this is what enables us, for example, to compute expected values of random quantities which depend on on multiple interrelated parameters.

Prerequisites

You must have passed 21-112 or 21-120. The following material will be assumed:

- **Functions of a single variable.** Polynomial, trigonometric, hyperbolic, exponential and logarithmic functions; domain and range; composition of functions; curve-sketching.
- **Elementary calculus.** Limits; continuity; single-variable differentiation and integration; chain, product and quotient rules; integration by substitution and by parts; logarithmic and implicit differentiation; convexity and concavity.

Please practice these skills if they're rusty. Since this is a summer course there will be little time for review, and I will assume that you are comfortable with this material.

Learning objectives

Upon successful completion of this course you should be able to:

- Compute and geometrically interpret the dot and cross products of vectors;
- Find the vector equation of a line and a plane in three-dimensional space;
- Perform elementary matrix algebra: matrix multiplication, determinants, inverses, rank;
- Interpret and solve systems of equations with vectors and matrix methods;
- Compute the the tangent plane to a surface of the form $z = f(x, y)$;
- Classify maxima and minima of surfaces of the form $z = f(x, y)$ using the gradient vector;
- Apply the method of Lagrange multipliers to solve optimization problems;
- Integrate multivariate functions over regions of $\mathbb{R}^2$ and $\mathbb{R}^3$.

Expectations

I expect you to: be physically and mentally present in lectures; do the homework and quizzes to the best of your ability; ask and answer questions in class; and abide by my and CMU's policies (see below). Also, **please ask for help if you need it**! This is a fast-paced summer course—you can't afford to fall behind, and I'm more than willing to help every last one of you understand the material.

You can expect me to: turn up to class on time and give high-quality lectures; be enthusiastic and respectful; assess homeworks, quizzes and tests fairly; provide regular feedback on the class's performance; and hold regular office hours (TBA).

2 Course content

Texts.

- *Calculus: Early Transcendentals* (7^{th} ed.) by James Stewart (§§12, 14, 15);
- *Introduction to Mathematical Programming* (4^{th} ed.) by Russell Walker (§§2, 6, 7).

List of topics.

- **Vectors and matrices.** Euclidean space: vectors, dot product, norm, projection onto a vector, cross product, span, linear independence; lines and planes; matrices, determinant and inverse of a matrix, rank; systems of equations. [2 weeks]
- **Optimization.** Multivariate functions, partial derivatives, chain rule; surfaces and tangent planes; gradient vector (∇) and extrema on surfaces, Hessian matrix; Lagrange multipliers, Karush–Kuhn–Tucker theorem. [$2\frac{1}{2}$ weeks]
- **Integration.** Double integrals over regions in $\mathbb{R}^2$; iterated integrals; triple integrals over regions in $\mathbb{R}^3$; polar, cylindrical and spherical coordinates; applications. [$1\frac{1}{2}$ weeks]

Schedule. What follows is an approximate schedule for the course and is subject to change.

Week	Date	Topic	Book and Section
1	19th May	Introduction and review	—
	20th May — 23rd May	Vectors	St 12.1–12.5, Wa 2.2–2.3
2	27th May — 29th May	Matrices	Wa 2.4–2.8
	30th May	**Exam 1**	—
3	2nd June — 4th June	Partial differentiation	St 14.1–14.5
	5th June — 6th June	Gradient vector	St 14.6–14.7
4	9th June — 11th June	Unconstrained optimization	Wa 6.1–6.6
	12th June — 13th June	Lagrange multipliers	St 14.8, Wa 7.1–7.5
5	16th June — 17th June	Lagrange multipliers	St 14.8, Wa 7.1–7.5
	18th June	**Exam 2**	—
	19th June — 20th June	Double integrals	St 15.1–15.3
6	23rd June	Applications to probability	St 15.5
	24rd June — 25th June	Triple integrals	St 15.7
	26th June	Other coordinate systems	St 10.3, 15.4, 15.8–15.9
	27th June	**Exam 3**	—

3 Assessment

This course will be assessed by means of **homework**, **quizzes** and a three **examinations**, details of which are given below.

- **Homework** (25%). Homework will be due in class every Tuesday and Friday, except the last Friday. Your lowest-scoring homework will be dropped.
- **Quizzes** (25%). There will be a short (∼10-minute) quiz at the beginning of class on Mondays, Wednesdays and Fridays, except the first Monday and the last Friday. Your two lowest-scoring quizzes will be dropped.
- **Examinations** (50%). There will be three exams, one for each section of the course. Your two highest-scoring exams will be worth 20%, and the third will be worth 10%.

The grade boundaries will be decided when all assessment is complete; the highest they will be is

A: 90% B: 80% C: 70% D: 60%

These grade boundaries might go down, but they will not go up. I will not grade on a curve.

4 Policies

Academic honesty: collaboration, cheating, plagiarism

My stance on academic honesty is fairly simple: **all work you submit should be your own work and should reflect your understanding**. If you're ever in need of additional help and are in any doubt about whether you're going to break any rules accidentally, come see me for help instead.

- **Collaboration.** You may talk to other people in the class about your homework questions, but any records you make during the discussion must be destroyed before permanent records are made. Any permanent records you have of your discussion should be made separately and independently. If you have collaborated then you should declare who you worked with.

  You will **not** be penalized for declared collaboration! However, undeclared collaboration is plagiarism, and collaboration outside of the above guidelines is considered cheating.

  For example, if you write anything on a whiteboard and your friend writes things down on paper, you and your friend should erase the whiteboard and throw away the paper before writing anything permanent, such as written-up solutions or rough work that you keep.

- **External resources.** You may use external resources, such as textbooks and the internet, as long as it is in such a way that you enhance your understanding and assessed material isn't trivialized by your use of such resources. If you use an external resource (i.e. anything which is not the course text) then you should cite your source: book title and author, web page URL, etc.

  You will **not** be penalized for declared use of external resources! However, undeclared use of external resources is plagiarism, and use of external resources outside of the above guidelines is considered cheating.

  For example, if you're stuck on a problem, find a solution on the internet and then paraphrase it in your answer, this *is not permitted* and constitutes cheating. However, if you're stuck on a problem, read up on the underlying theory and then work out a solution by yourself, this *is permitted* so long as you cite your source.

Attendance

Since this is such a fast-paced course, I expect you to attend all lectures. I will use lecture time not only to teach the material, but also to distribute resources, collect and return homework, administer quizzes and communicate important information about the course. There is no 'attendance grade' and I cannot force you to attend class, but you skip class at your own peril.

Late homework

It is *your* responsibility to ensure that I receive homework when it's due. If there are any special circumstances, such as illness or planned absence, you should let me know as soon as possible. (The sooner you ask for an extension, the more likely I am to grant it.)

Other issues

Accommodations. Please ensure I'm aware of any accommodations that need to be made, such as extra time on quizzes and the exam, or large text, and give supporting documentation from the Office of Disability Resources.

Missing an exam. If you know in advance that you will miss an exam, you should inform me as soon as you know so that it can be re-scheduled.

In-test materials. Your brain is the only thing containing any significant mathematical content that you have on your person during the exams. You may not use notes or calculators.