

21-256 Homework 8

Due Thursday 26th June 2014

... but if you want feedback before the test, you should submit it on Wednesday

1. Compute $\int_0^2 \int_0^4 y^3 e^{2x} dy dx$ and verify that it is equal to $\int_0^4 \int_0^2 y^3 e^{2x} dx dy$.
2. Find the average value of the function $f(x, y) = \frac{1+x^2}{1+y^2}$ on the rectangle R defined by $0 \leq x \leq 1$ and $0 \leq y \leq \sqrt{3}$.
3. Compute $\int_0^1 \int_{x^2}^x (1+2y) dy dx$ and find a region U of the xy -plane such that this integral is equal to $\int_U (1+2y) dA$.
4. Find the average value of x^3 on the circle in the xy -plane with center $(1, 1)$ and radius 2.
5. Compute $\int_U y dA$, where U is the region of the xy -plane bounded by the curves $y = x - 2$ and $x = y^2$.
6. Find the region bounded by the coordinate planes (i.e. the xy -, yz - and xz -planes) and the plane $3x + 2y + z = 6$.
- †7. The joint density function of a pair of random variables X and Y is given by

$$f(x, y) = \begin{cases} kx(1+y) & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of the constant k , and compute $\mathbb{P}(X \leq 1, Y \leq 1)$ and $\mathbb{P}(X + Y \leq 1)$.

- †8. It's 1996 and cell phones aren't cool yet. Two students, Xavier and Yolanda, agree to meet at noon each day in a coffee shop, but typically they're both a bit late. Their arrival times are independent of each other (because it's 1996 and cell phones aren't cool yet). Xavier arrives at X minutes after noon, and Yolanda arrives at Y minutes after noon.

Xavier is usually not very late but could arrive any time after noon; Yolanda always arrives by 12:10 but is more likely to be late than prompt. As such, they model their lateness with the following probability density functions:

$$f_X(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad f_Y(y) = \begin{cases} \frac{y}{50} & \text{if } 0 \leq y \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

Yolanda is willing to wait up to half an hour for Xavier, after which time she'll leave and write him a strongly worded letter (because it's 1996 and cell phones aren't cool yet).

What is the probability that they meet?

† Questions 1–6 are worth 10 points each; Questions 7–8 are worth 20 points each.