21-256 Homework 6

Updated Tuesday 10th June 2014
Due Friday 13th June 2014

1. Find ∇f when $f(x,y) = \ln(x^6 + 3x^2y^4 + y^6)$.

2. Find ∇g when $g(x_1, x_2, \cdots, x_n) = x_1 + 2x_2^2 + 3x_3^3 + \cdots + nx_n^n$.

3. Find and classify the critical points of the function $f(x,y) = xy(1-x-y)$.

4. Find the absolute maximum and minimum values of the function $g(x,y) = 4x+6y-x^2-y^2$ on the subset of $\mathbb{R}^2$ defined by $0 \leqslant x \leqslant 4$ and $0 \leqslant y \leqslant 5$.

5. The base of a cuboidal aquarium with given volume V is made of slate and the sides are made of glass. If slate costs of five times as much (per unit area) as glass, find the dimensions of the aquarium that minimize the cost of the materials.

6. Determine whether the matrices A and B given below are positive-definite, negative-definite or neither.

$$A = \begin{pmatrix} 4 & -1 & 3 \\ -1 & 2 & 0 \\ 3 & 0 & 5 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & -4 & -1 \\ -4 & 2 & 3 \\ -1 & 3 & 5 \end{pmatrix}$$

7. Find and classify the critical points of the function

$$g(x,y,z) = 2x^2 + xz + y^2 + yz + z^2 - 6x - 8y - 7z + 12$$

8. Let q be a function defined by $q(x,y,z) = \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + \mathbf{a}\cdot\mathbf{x}$, where

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}, \qquad Q = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Show that $H_q = Q$.

9. Find a function f such that $\nabla f(0,0,0) = \mathbf{0}$ and $H_f(0,0,0) = \begin{pmatrix} 3 & 0 & 2 \\ 0 & 2 & 2 \\ 2 & 2 & 5 \end{pmatrix}$, and use this information to determine whether $(0,0,0)$ is a local maximum or local minimum of f.

10 points will be awarded for submitting the homework on time.

Extra credit problems

E1. Let q be a function defined by $q(x,y,z) = \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + \mathbf{a}\cdot\mathbf{x}$, where $\mathbf{x} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\mathbf{a}$ is any 3-dimensional vector, and Q is any symmetric 3×3 matrix. Show that $H_q = Q$.