21-256 Homework 5

Due Friday 6th June 2014

1. Find and sketch the largest possible domain of the bivariate function f defined by

$$f(x,y) = \frac{1}{\sqrt{x^2 - y}}$$

2. Compute $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial^2 f}{\partial x^2}$ and $\dfrac{\partial^3 f}{\partial x \partial y \partial z}$ when $f(x,y,z) = \sqrt{x^2 + y^2 + z^2 + xyz}$.

3. Find the partial derivatives of $u(x,y) = \dfrac{ax + by}{cx + dy}$, and find $\dfrac{dy}{dx}$ when $u(x,y) = 1$.

4. Compute $\dfrac{dz}{dt}$ when $z = e^{xy^2}$, where $x = 3t + 4\sin t$ and $y = \tan t + 5$.

5. Let $g(x,y) = 2y \sin x + 3x \cos y$, and suppose that $x = uvw$ and $y = uv + vw + wu$. Compute the partial derivatives of g with respect to u, v and w.

6. In 2014, global wheat production is expected to be 701.7 million metric tons, global rainfall is expected to be 990 mm, and the average global temperature is expected to be 14.4°C. It is also estimated that global rainfall is decreasing at a rate of 1 mm/year and the average temperature is rising at a rate of 0.018°C/year. Let W denote the global production of wheat in a given year (in millions of metric tons), let T denote the average global temperature over the year (in °C) and let R denote the average rainfall over the year (in mm). At current production levels, it is estimated that $\frac{\partial W}{\partial T} = -2$ and $\frac{\partial W}{\partial R} = 0.8$.

Model W as a function of T and R. Does your model predict that wheat production will increase or decrease this year? At what rate?

7. Show that if x, y, z are related implicitly by $x^2y + y^2z + z^2x + \sin(xy) = 0$ then $\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x} = -1$.

8. Find the vector equation of the tangent plane to the surface $f(x,y,z) = k$ at the point (a,b,c). What does this tell you about the relationship between ∇f and the tangent plane?

9. Find the linearization of the function $g(x,y) = ye^{x \sin(xy)} - x$ at the point $(-1,2)$.

10. Find the linear approximation of the function $h(x,y) = 1 - xy\cos(\pi xy)$ at the point $(1,1)$ and use it to approximate $h(1.01, 0.97)$.

Extra credit problems

E1. Suppose $x_1, x_2, \cdots, x_n$ are related implicitly by $F(x_1, x_2, \cdots, x_n) = k$ for some constant k and function F of n variables. Using the chain rule, show that

$$\frac{\partial x_1}{\partial x_2} \cdot \frac{\partial x_2}{\partial x_3} \cdot \cdots \cdot \frac{\partial x_{n-1}}{\partial x_n} \cdot \frac{\partial x_n}{\partial x_1} = (-1)^n$$