

# 21-256 Homework 4

Due Monday 2nd June 2014 (extra credit for early submission, see below)

1. Show that if  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  are linearly dependent vectors in  $\mathbb{R}^3$  then  $[\mathbf{u}, \mathbf{v}, \mathbf{w}] = 0$ .

2. Compute  $A + B$ ,  $2A^T - 3B$ ,  $AB$  and  $BA$  when  $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$ .

3. Compute the determinants of the following matrices:

$$A = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 4 \\ -1 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

4. Compute the determinant of the following matrix:

$$\begin{pmatrix} 1 & -5 & 2 & 1 \\ -1 & 0 & 0 & 2 \\ 0 & 4 & 0 & 0 \\ 3 & -2 & 1 & 1 \end{pmatrix}$$

5. Find  $A\mathbf{u}$ ,  $A\mathbf{v}$  and  $A\mathbf{w}$  when

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix}$$

6. For each matrix from Question 3, either find its inverse or say why it doesn't exist.

7. Show that  $\det \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ 0 & b_2 & c_2 & d_2 \\ 0 & 0 & c_3 & d_3 \\ 0 & 0 & 0 & d_4 \end{pmatrix} = a_1 b_2 c_3 d_4$ .

8. Solve the following systems of equations. Use a parameter  $\lambda$  if necessary.

$$(a) \begin{cases} 2x + y + z = 3 \\ x + 2y + z = -1 \\ x + y + 2z = 2 \end{cases} \quad (b) \begin{cases} 2x + y + z = 3 \\ x + 2y + z = -1 \\ 3x + 3y + 2z = 2 \end{cases}$$

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Each question is worth 10 points. You will receive 10 points for attempting all questions, and 10 points for handing your homework in on time. If you submit your homework in class on Friday, you'll receive 10 points of extra credit (up to a maximum of 100 points).