21-256 Homework 3

Due Wednesday 28th May 2014

1. Use the scalar triple product to verify that following three vectors are coplanar:

$$\mathbf{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}, \quad \mathbf{v} = 3\mathbf{i} - \mathbf{j}, \quad \mathbf{w} = 5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$$

2. Find the acute angle between the lines $2x - y = 3$ and $3x + y = 7$ in $\mathbb{R}^2$.

3. Find the two unit vectors (in $\mathbb{R}^2$) that are parallel to the tangent line to the curve $y = x^2$ at the point $(2, 4)$.

4. Find the vector equation of the line through the point $(0, 14, -10)$ which is parallel to the line $\mathbf{r} = (-1 + 2\lambda)\mathbf{i} + (6 - 3\lambda)\mathbf{j} + (3 + 9\lambda)\mathbf{k}$.

5. Find the vector equation of the line of intersection of the planes $x + 2y + 3z = 1$ and $x - y + z = 1$.

6. Find the vector equation and the linear equation of the plane through the origin and the points $(2, -4, 6)$ and $(5, 1, 3)$.

7. Find the point at which the following lines intersect, and find an equation of the plane containing both lines:

$$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

8. Express the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ as a linear combination of the two vectors $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

9. Show that the set $\left\{ \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \end{pmatrix} \right\}$ is linearly independent.

10. Show that the set $\{2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}, -3\mathbf{i} + 2\mathbf{j} + \mathbf{k}, -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}\}$ is linearly dependent.

Extra credit problems

E1. Two parallel planes have equations

$$ax + by + cz + d_1 = 0 \quad \text{and} \quad ax + by + cz + d_2 = 0$$

Prove that the distance between the two planes is $\dfrac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.