21-256 Exam 1: Vectors and Matrices (sample)

Monday 2nd June 2014, 10:30–11:20am

Instructions (please read carefully)

This test is split into three sections of four questions each:

- **Section A:** Vector algebra
- **Section B:** Lines and planes
- **Section C:** Matrix algebra

You should attempt **exactly ten** questions, including **at least three** questions from each section. All questions will be marked out of 10. The duration of the test is 50 minutes. Please write legibly in the blue book provided. Calculators and other electronic devices are not permitted.

Please indicate on the table below which questions you attempted.

Question	Attempted? (✓)	Score
A1		
A2		
A3		
A4		
B1		
B2		
B3		
B4		
C1		
C2		
C3		
C4		
Total	—	

Name: _____

Section A

A1. Find $\|2\mathbf{v} - \mathbf{w}\|$ and $\mathbf{v} \times \mathbf{w}$, where $\mathbf{v} = \mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} - \mathbf{k}$.

A2. Find $[\mathbf{a}, \mathbf{b}, \mathbf{c}]$ when $\mathbf{a} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$.

A3. Compute the scalar and vector projections of $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ onto $\begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix}$.

A4. Find the area of the triangle with vertices $(1, 0, 2)$, $(0, 2, 1)$ and $(3, 3, 3)$.

Section B

B1. Find vector equation of the plane in $\mathbb{R}^3$ which passes through the points $(1, 0, 1)$, $(3, 0, 1)$ and $(0, -1, -2)$.

B2. Find the acute angle between the line $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ and the line of intersection of the planes $x + y + z = 2$ and $x + z = 0$.

B3. Find the distance from the plane $3x + y - z = 2$ to the origin.

B4. Find the acute angle between the yz-plane and the plane $x + z = 1$.

Section C

C1. Find nonzero 2×2 matrices A and B such that $AB = 0$.

C2. Find the determinant and inverse of $\begin{pmatrix} -2 & 4 \\ -6 & 3 \end{pmatrix}$.

C3. Find the determinant of $\begin{pmatrix} 1 & 0 & 1 \\ -7 & 3 & 6 \\ 1 & 0 & -3 \end{pmatrix}$.

C4. Using determinants, show that the vectors $\mathbf{i} - \mathbf{j}$, $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$ are linearly independent.

Please submit this question sheet with your answer booklet.