21-128 and 15-151 problem sheet 7

Solutions to the following five exercises and optional bonus problem are to be submitted through blackboard by 8:30AM on

Thursday 3rd November 2016.

Problem 1

How many ways are there to pick two cards from a standard 52-card deck, such that the first card is a spade and the second card is not an ace?

Problem 2

Count the number of hands of six cards from a standard deck of 52 cards that contain at least one card of every suit.

Problem 3

Find the number of functions $f : [6] \to [6]$ such that f contains exactly three elements in its image.

Problem 4

We roll a fair six-sided die exactly four times. For $k \in \{0, 1, 2, 3, 4\}$, determine the probability that we roll a six exactly k times. Check your answer by verifying that these probabilities sum to one.

Problem 5

By counting in two ways, prove that $\sum_{k=1}^{n} 2^{k-1} = 2^n - 1$ for all $n \in \mathbb{N}$.

Bonus Problem - (2 points)

Tram tickets have six-digit numbers (from 000000 to 999999). A ticket is called *lucky* if the sum of its first three digits is equal to the sum of its last three digits. A ticket is called *medium* if the sum of all its digits is 27. Let A and B denote the numbers of lucky tickets and medium tickets respectively. Prove that $A = B$.