

Detecting a regime change & connections to mathematical finance

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Abstract

The topic of statistical surveillance is presented. We begin by defining the out-of-control and in-control states of the process and describe how we can distinguish them by using statistics based on the observations of the process. In this context we present the CUSUM statistic. As an example of a problem in statistical surveillance we will present the topic of change-point detection in the Brownian motion model with multiple alternatives. In this problem, the objective is to detect a change in the constant drift of a Brownian motion by means of a stopping rule that reacts fairly fast (with respect to a carefully selected criterion) to an abrupt change but at the same time keeps the frequency of false alarms above a certain threshold. In this context we also examine the specific instance of two-sided alternatives, where the two-sided CUSUM stopping rule (2-CUSUM) is employed. We find the best 2-CUSUM stopping rule. We finally show very strong asymptotic properties of carefully chosen 2-CUSUM stopping rules. We proceed to show connections of the 2-CUSUM process to two very important processes in finance the drawdown process and the rally process. We address the question of computing the probability that a drop of $a\%$ from the running maximum of the underlying asset precedes a rally of $b\%$ from the running minimum of the asset, under the assumption of the geometric Brownian motion dynamics governing the underlying asset. We conclude by discussing some open problems related to the above topics.