Review

JV Practice 10/18/20

C.J. Argue

1 Set 1

1. A function f is defined by $f(z) = i\overline{z}$, where $i = \sqrt{-1}$ and $\overline{z}$ is the complex conjugate of z. How many values of z satisfy both $|z| = 5$ and $f(z) = z$?
2. Triangle ABC has $AB = 25$, $AC = 20$, and $BC = 15$. Point D is on $\overline{AB}$ such that $AD = 10$. Compute CD.
3. The quadratic equation $x^2 + mx + n$ has roots twice those of $x^2 + px + m$, and none of m, n, and p is zero. What is the value of n/p?
4. Circle ω has radius 5. Points A, B, C lie on circle ω such that $\triangle ABC$ is an isoceles triangle with $\angle B = 30$. Compute the area of $\triangle ABC$.

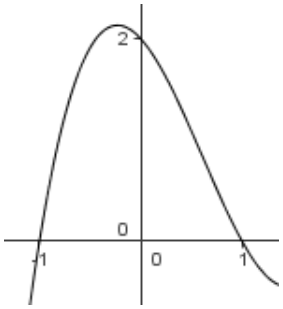
Set 2

1. If $3^{\tan(x)} = 81^{\sin(x)}$, compute all possible values of $\cos(x)$.
2. In $\triangle ABC$, $AB = 6$, $BC = 8$, and $AC = 10$. Points M and N are on BC and AC, respectively, and MN intersects the angle bisector of $\angle C$ at P. If $MP = 2$ and $PN = 5$, compute the area of $\triangle MNC$.
3. The graph of the polynomial $P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ has five distinct x-intercepts, one of which is at $(0, 0)$. Which of the coefficients (a, b, c, d, or e) cannot be 0?
4. Find the number of ordered pairs of real numbers (a, b) such that $(a + bi)^{2020} = a - bi$.
5. In $\triangle ABC$, $\angle A = 45°$, $\angle B = 60°$ and $AC = \sqrt{15}$. Point D lies on $\overline{AB}$ such that $\overline{AB}$ and $\overline{CD}$ are perpendicular. The circle with diameter $\overline{AB}$ intersects $\overline{CD}$ at E. Compute DE^2.
6. The equation $2^{333x-2} + 2^{111x+2} = 2^{222x+1} + 1$ has three real roots. Compute the sum of the roots.

Set 3

1. Suppose that the real part of the complex number z is equal to 1 and the real part of z^2 is equal to -2. Compute the real part of z^3.
2. Square $ABCD$ has side length 4. Equilateral triangles ABE and BCF are constructed on the exterior of square $ABCD$. Compute the area of $\triangle DEF$.
3. Points P, Q, R lie inside $\triangle ABC$ such that P lies on $\overline{AR}$, R lies on $\overline{CQ}$, and Q lies on $\overline{BP}$. Given that $AP = CR = BQ = 4$ and $PR = RQ = QP = 3$, compute the area of $\triangle ABC$.

4. Part of the graph of $f(x) = ax^3 + bx^2 + cx + d$ is shown. What is b?



5. The roots of the polynomial $x^6 - 12x^5 + ax^4 + bx^3 + cx^2 + dx + e$ form a geometric progression. If the sum of the reciprocals of the roots is 6, compute e.

6. For how many positive integers n less than or equal to 1000 is $(\sin t + i \cos t)^n = \sin nt + i \cos nt$ true for all real t?