

Geometry inequalities 2

1. Warm-up

1. In triangle ABC , we know that $\angle A > \angle B$. Prove that $BC > \frac{1}{2}AB$.
2. From the point M , which does not belong to the line a , the perpendicular MD is drawn, and let A and B are some points on line a (that are different from point D) such that point D belongs to the segment AB . Prove that if $DA > DB$, then $MA > MB$.

2. Problems

1. The perpendicular bisector to the side BC of $\triangle ABC$ intersects side AB at point D , and the continuation of side AC beyond point A at point E . Prove that $AD < AE$.
2. An angle-bisector BD is drawn in the $\triangle ABC$. Prove that $AB > AD$.
3. The peninsula has an acute-angle shape, inside which Dan build his house. How does Dan, starting from his house, get to the one coast of the peninsula, then to the another and return home, passing along the shortest path?
4. On the sides AB and AC of $\triangle ABC$, choose points P and Q so that $PB = QC$. Prove that $PQ < BC$.
5. Prove that the area of convex quadrilateral $ABCD$ doesn't exceed $\frac{1}{2}(AB \cdot BC + AD \cdot DC)$.
6. Prove that in any triangle, the smaller side corresponds to the smaller median.

*Looking forward to hear from you soon!
Have fun!*

