Pigeonhole Principle

JV Practice 5/10/20

Elizabeth Chang-Davidson

1 Warmup

1. (Classic problem) I have pink, red, orange, yellow, green, blue, and purple socks. How many socks do I need to take out of my sock drawer to guarantee a matching pair if I am too sleepy to open my eyes?

2. There are 82 colored cubes. Prove that you can either pick ten cubes of different colors or ten cubes of the same color.

3. Prove that for any set of five integers, there are three integers whose sum is divisible by 3. (Hint: try some examples if you get stuck)

2 Problems

1. If I am splitting up 39 students into 9 breakout rooms, what is the smallest size the largest breakout room could be?

2. During the last week Elizabeth ate 15 chocolate bars. Prove that during one of the days she ate at least three chocolate bars.

3. 100 books are delivered to students so they can do their homework at home. What is the maximum number of students such that every student could get a different number of books?

4. 8 different positive integers are given, all at most 15. Prove that among all pairwise differences of those numbers at least three are the same.

5. There are 52 people in a room. what is the largest value of n such that the statement "At least n people in this room have birthdays falling in the same month" is always true?

6. Several distinct positive integers are randomly chosen between 1 and 2006, inclusive. What is the number of integers needed to guarantee that for some group of three of these integers, the three in the group all pairwise have differences that are multiples of 5?

7. Show that given any 9 points inside or on a square of side length 1 we can always find 3 that form a triangle with area less than or equal to $\frac{1}{8}$.

8. Any five points are taken inside or on a square with side length 1. Let a be the smallest possible number with the property that it is always possible to select one pair of points from these five such that the distance between them is equal to or less than a. What is a?

9. What is the smallest n such that if a $3 \times n$ grid of points is colored red and blue, there will always be a rectangle whose 4 corners have the same color?

10. We select 38 even positive integers, all less than 1000. What is the minimum integer d such that you are guaranteed two of the integers have a difference less than d?

11. Prove that if you choose 51 integers from the set $\{1, 2, \ldots, 100\}$ at least one of them must be divisible by another.

3 Challenge Problems

1. Let the sum of a set of numbers be the sum of its elements. Let S be a set of positive integers, none greater than 15. Suppose no two disjoint subsets of S have the same sum. What is the largest sum a set S with these properties can have?

2. Each of eight boxes contains six balls. Each ball has been colored with one of n colors, such that no two balls in the same box are the same color, and no two colors occur together in more than one box. Determine, with justification, the smallest integer n for which this is possible.

3. CMU's Scotty dog colors every point on the plane either red or black. Prove that no matter how the coloring is done, there must exist two points, exactly a mile apart, that are the same color.