Sequences and Series II

JV Practice 5/3/20

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1 Week 2 Warm-Ups

1. Evaluate the sum
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{2019\cdot 2020}$$

2. Evaluate the sum
$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{99}+\sqrt{100}}.$$

2 Week 2 Problems

1. Find the value of $a_2+a_4+\cdots+a_{98}$ if a_n is an arithmetic progression with common difference 1 and $a_1+a_2+\cdots+a_{98}=137$.

2. Evaluate the sum
$$\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+dots+\frac{1}{97\cdot 100}.$$

3. Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N?

4. Evaluate the sum
$$\frac{1}{3^2+1}+\frac{1}{4^2+2}+\frac{1}{5^2+3}+\dots$$

5. Let $a<b<c$ be three integers such that a,b,c is an arithmetic progression and a,c,b is a geometric progression. What is the smallest possible value of c?

6. Evaluate the sum
$$\left(\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\right)+\left(\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+\dots\right)+\left(\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\dots\right)+\dots$$

7. The sequence (a_n) satisfies $a_1=1$ and
$$5^{(a_{n+1}-a_n)}-1=\frac{1}{n+\frac{2}{3}}$$
for $n\geq 1$. Let k be the least integer greater than 1 for which a_k is an integer. Find k.

8. Evaluate the sum
$$\frac{3}{1^2\cdot 2^2}+\frac{5}{2^2\cdot 3^2}+\frac{7}{3^2\cdot 4^2}+\cdots+\frac{29}{14^2\cdot 15^2}.$$

9. Define the Fibonacci sequence (F_n) by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$. Evaluate the sum

$$T = \frac{F_1}{5} + \frac{F_2}{25} + \frac{F_3}{125} + \frac{F_4}{625} + \frac{F_5}{3125} + \cdots$$

10. Evaluate the sum

$$\sum_{n=1}^{\infty} \frac{1}{n^3 + 3n^2 + 2n}$$