JV Review

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1 Review Problems

1. [**AMC 10 2008**] For a positive integer n, $\langle n\rangle$ denote the sum of all positive divisors of n with the exception of n itself. For example, $\langle 4\rangle = 1+2 = 3$. What is $\langle\langle\langle 6\rangle\rangle\rangle$?

2. Suppose $2n = p_1 + p_2$, where p_1 and p_2 are two consecutive prime numbers. Prove that n is composite.

3. [**AMC 10 2009**] Each morning of her 5-day week, Jane bought either a 50-cent muffin or a 75-cent bagel. Her total cost at the end of the week was a whole number of dollars. How many muffins did she buy?

4. [**CMIMC 2018**] If a, b, c are relatively prime integers such that

$$\frac{a}{b+c} = 2 \qquad \text{and} \qquad \frac{b}{a+c} = 3$$

then what is $|C|$?

5. Let a, b, c be integers such that both $ab + 9b + 81$ and $bc + 9c + 81$ are divisible by 101. Prove that $ca + 9a + 81$ is also divisible by 101.

6. What is the reminder when $3^0 + 3^1 + 3^2 + \cdots + 3^{2019}$ is divided by 10?

7. [**AMC 10 2009**] What is the reminder when $3^0 + 3^1 + 3^2 + \cdots + 3^{2019}$ is divided by 8?

8. [**Purple Comet 2011**] What is the smallest prime that does not divide $9 + 9^2 + \cdots 9^{2019}$?

9. [**AMC 10 2008**] Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

10. If a and b are positive integers relatively prime to m with $a^x \equiv b^x \pmod{m}$ and $a^y \equiv b^y \pmod{m}$ then prove that $a^{\gcd(x,y)} \equiv b^{\gcd(x,y)} \pmod{m}$.

11. If p and $p^2 + 2$ are both primes, then prove that $p^3 + 2$ is also a prime.

12. If $2n + 1$ and $3n + 1$ are perfect squares for $n > 0$, then show that $5n + 3$ is not a prime.

13. [**Purple Comet 2011**] Find the prime p such that $71p + 1$ is a perfect square.

14. [**CMIMC 2018**] Find all integers n such that $(n-1)\cdot 2^n + 1$ is a perfect square.