Fermat's Little Theorem Practice

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Problems

1. Find 3^{31} mod 7.
2. Find 2^{35} mod 7.
3. Find 128^{129} mod 17.
4. (1972 AHSME 31) The number 2^{1000} is divided by 13. What is the remainder?
5. Find 29^{25} mod 11.
6. Find $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60}$ mod 7.
7. Let

$$a_1 = 4 \text{ , } a_n = 4^{a_{n-1}} \text{ , } n > 1$$

   Find a_{100} mod 7.

8. Solve the congruence

$$x^{103} \equiv 4 \text{ mod } 11.$$

9. Find all integers x such that $x^{86} \equiv 6$ mod 29.
10. What are the possible periods of the sequence $x, x^2, x^3, ...$ in mod 13 for different values of x? Find values of x that achieve these periods.
11. If a googolplex is $10^{10^{100}}$, what day of the week will it be a googolplex days from now? (Today is Sunday)
12. Suppose that p and q are distinct primes, $a^p \equiv a \pmod{q}$, and $a^q \equiv a \pmod{p}$. Prove that $a^{pq} \equiv a \pmod{pq}$.
13. Find all positive integers x such that $2^{2^x+1} + 2$ is divisible by 17.
14. An alternative proof of Fermat's Little Theorem, in two steps:
    (a) Show that $(x + 1)^p \equiv x^p + 1 \pmod{p}$ for every integer x, by showing that the coefficient of x^k is the same on both sides for every $k = 0, ..., p$.
    (b) Show that $x^p \equiv x \pmod{p}$ by induction over x.
15. Let p be an odd prime. Expand $(x - y)^{p-1}$, reducing the coefficients mod p.