Math 820 A   Fall 2021

Advanced Topics in Analysis: Collective Dynamics and Structure in Infinite-Dimensional Systems

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Course Description: This course will survey recent progress regarding the emergence of coherent scaling behavior in important PDE and kinetic models, focusing on phase transitions, aggregation, and random shock clustering. We deal with models at various levels of microscopic detail: diffuse-interface, sharp-interface, particles, kinetic equations, stochastic models. Dynamical concepts motivated by the theory of stable laws and infinite divisibility in probability play a significant role.

Prerequisites: Useful background includes PDE, Measure & Integration, Probability, Functional analysis, Sobolev spaces. Not all are required, but at least two semesters of graduate study in these subjects is recommended.

Objectives: The aim of the course is for you to learn about a number of quite different kinds of mathematical models of collective dynamics and pattern evolution in PDEs, and the mathematical methods (both rigorous and non-rigorous) involved in their analysis. We seek to understand how models relate to each other by proceeding to larger time and space scales. Several of the models that we deal with (e.g., diffuse and sharp interface models) have become important across applied mathematics quite broadly, as they have been adapted to study quite unrelated phenomena. Along the way you will study a variety of mathematical methods, both rigorous and non-rigorous, that have proved effective for explaining universal dynamical behavior exhibited by these systems.

Evaluation. The grade will be based on a course project, consisting of two parts: a written report (at least 6 pages in length) and an oral presentation (20 minutes plus time for questions). A list of suggested topics and guidance regarding the project will be supplied. The goal is to explore published research on a topic and explain the main ideas to your peers.

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Preliminary outline of topics

- Pattern formation, model hierarchies, and large-scale behavior in 1D. Allen-Cahn equation. Metastability, domain-wall interactions. “1D bubble bath.” A mean-field coarsening model and Gallay & Mielke’s remarkable solution.
- Multi-D interface formation and evolution. Korteweg’s nonlocal approach to interfacial energy. Degenerate Cahn-Hilliard phase separation model. Monopole model. Lifshitz-Slyozov-Wagner kinetic equation. Kohn-Otto coarsening rate estimates.
- Mean-field models of clustering and aggregation: Smoluchowski’s coagulation equations. Scaling dynamics with solvable kernels: self-similar solutions, domains of attraction, scaling attractor, universal limits.
- Dynamics of random shock-wave fields. Evolution from Sisyphean initial data (Lévy processes with only downward jumps) by Smoluchowski’s equation. Menon’s formal treatment of Markov shock-wave fields. Relation to the Shandarin-Zel’dovich cosmology model.
- Branching processes. Clustering dynamics of clans. Scaling limits of Galton-Watson processes, continuous-state branching processes.

Methods and structures to be developed and explored

Gradient flow dynamics. Model reduction by constrained kinematics. Matched asymptotic analysis of sharp-interface limits. Gamma limits of gradient flows. Center manifold reduction.

Tools for dynamic scaling analysis: Regular variation. Laplace and Bernstein transforms. Pick functions. Tauberian theorems. Renormalization.