Math 259, Winter 2009. Answers to Unit Test 3 Review Problems – Set A.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1. Try doing this problem as a Lagrange multiplier problem with $f(x, y, z) = x^2 + y^2 + z^2$ and constraints $g(x, y, z) = x + 2y + 3z - 3 = 0$ and $h(x, y, z) = x^2 + y^2 - z^2 = 0$.

1.(a) The closest point has coordinates $(x, y, z) = (0.2562, 0.5125, 0.5729)$.

1.(b) The furthest point has coordinates $(x, y, z) = (-1.7562, -3.5125, 3.9271)$.

2.(a) The Riemann sum is equal to 8.

2.(b) The Riemann sum is equal to -12.

2.(c) The Riemann sum is equal to $0.5\pi^2$. (Make sure your calculator is in RADIAN mode.)

3.(a) 0.

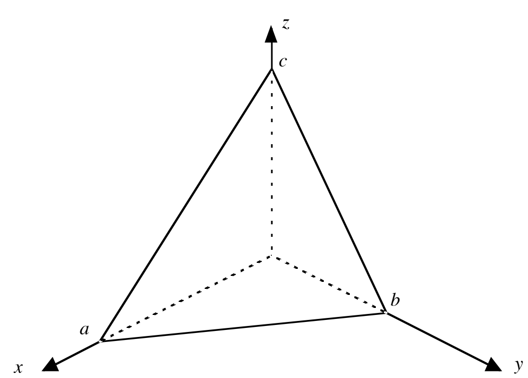
3.(b) 0.

3.(c) π.

3.(d) 1.

3.(e) 2.

4.(a) A plausible sketch of the solid is shown below. Your sketch may differ (but still be correct).



4.(b) Volume = $\displaystyle\int_0^a \int_0^{b-bx/a} c\left(1-\frac{x}{a}-\frac{y}{b}\right)dydx$.

4.(c) Volume = $(abc)/6$.

4.(d) Mass = $\displaystyle\int_0^a \int_0^{b-bx/a} \int_0^{c(1-x/a-y/b)} (x+y+z)\,dzdydx$.

4.(e) Mass = $abc(a+b+c)/24$.

5.(a) Global max = $\sqrt{5}$, attained at $(x, y) = (2\sqrt{5}/5, \sqrt{5}/5)$. Global min = $-\sqrt{5}$, attained at $(x, y) = (-2\sqrt{5}/5, -\sqrt{5}/5)$.

5.(b) Global max = 4, attained at $(x, y) = (\pm 2, 0)$. Global min = -4, attained at $(x, y) = (0, \pm 2)$.

5.(c) Global max = 3, attained at $(x, y) = (3\sqrt{2}/2, \sqrt{2})$ and $(x, y) = (-3\sqrt{2}/2, -\sqrt{2})$. Global min = -3, attained at $(x, y) = (-3\sqrt{2}/2, \sqrt{2})$ and $(x, y) = (3\sqrt{2}/2, -\sqrt{2})$.

5.(d) There is no global maximum. Global min = 18/7, attained at (x, y) = (9/7, 6/7. 3/7).

6.(a)
(I) Mass = a^3.
(II) x-coordinate = $7a/12$.
(III) y-coordinate = $7a/12$.

6.(b)
(I) Mass = 128/5.
(II) x-coordinate = 0.
(III) y-coordinate = 20/7.

6.(c)
(I) Mass = π
(II) x-coordinate = $\pi - 4/\pi$.
(III) y-coordinate = $\pi/8$.

6.(d)
(I) Mass = $\pi a^3/3$.
(II) x-coordinate = 0.
(III) y-coordinate = $3a/(2\pi)$.

6.(e)
(I) Mass = $2\pi/3 + \sqrt{3}/4$.
(II) x-coordinate = 0.
(III) y-coordinate = $\dfrac{36\pi + 33\sqrt{3}}{32\pi + 12\sqrt{3}}$.

7.(a) 310/3.

7.(b) $3\sqrt{2}$.

7.(c) 49/24.

7.(d) $(14\sqrt{14} - 1)/6$.

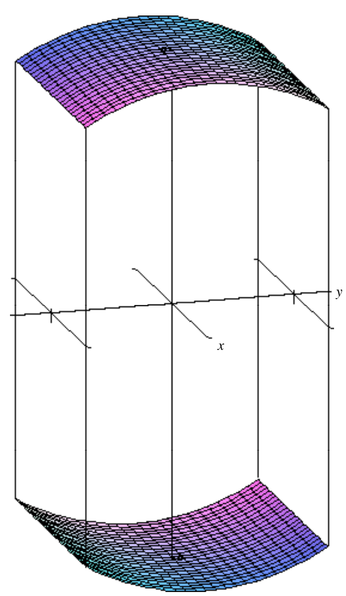
8.(a) Volume = $\pi a^2 h$.

8.(b) Volume = 2π.

8.(c) Volume = $\dfrac{\pi a^3}{3}\left(2 - \sqrt{2}\right)$.

8.(d) Volume = $\pi/4$.

9.(a) A sketch of the region is given below. This sketch is a little distorted in that the "shadow" cast in the xy-plane appears to be a rectangle – it should appear more like a square.



9.(b) Volume = $2\displaystyle\int_{-R/2}^{R/2}\int_{-R/2}^{R/2}\sqrt{R^2 - y^2}\,dydx$.

9.(c) Volume = $\dfrac{R^3}{6}\left(2\pi + 3\sqrt{3}\right)$. Try a trigonometric substiution.

10. Try using Lagrange multipliers with $f(x, y, z, \alpha)$ given by the area formula and constraints $g(x, y, z, \alpha) = x + y + z - P = 0$ and the Law of Cosines. The maximum area is achieved when $x = y = z = P/3$ and $\alpha = \pi/3$.