Math 122, Fall 2008. Answers to Unit Test 1 Review Problems – Set B.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) $\int s\cdot\sqrt{s+1}\cdot ds=\frac{2}{5}(s+1)^{\frac{5}{2}}-\frac{2}{3}(s+1)^{\frac{3}{2}}+C.$

1.(b) $\int\frac{x^2+x}{\sqrt{x+1}}\cdot dx=\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{2}{3}(x+1)^{\frac{3}{2}}+C.$

1.(c) $\int x^2\cdot\sqrt{x-2}\cdot dx=\frac{2}{7}(x-2)^{\frac{7}{2}}+\frac{8}{5}(x-2)^{\frac{5}{2}}+\frac{8}{3}(x-2)^{\frac{3}{2}}+C.$

1.(d) $\int\frac{p}{\sqrt{p+1}}\cdot dp=\frac{2}{3}(p+1)^{\frac{3}{2}}-2(p+1)^{\frac{1}{2}}+C.$

2.(a) $\int_0^2 xf'(x)dx=\left[x\cdot f(x)\right]_0^2-\int_0^2 f(x)dx=3.$

2.(b) $\int_2^4 f'(x)(2+3f(x))dx=2\cdot\left[f(x)\right]_2^4+\frac{3}{2}\left[f(x)^2\right]_2^4=-20.$

2.(c) $\int_0^2 f(3x)dx=\frac{1}{3}\int_0^6 f(u)du=\frac{10}{3}$ using the substitution $u = 3x$.

2.(d) $\int_0^2 x\cdot f(x^2)dx=\frac{1}{2}\int_0^4 f(u)du=2$ using the substitution $u = x^2$.

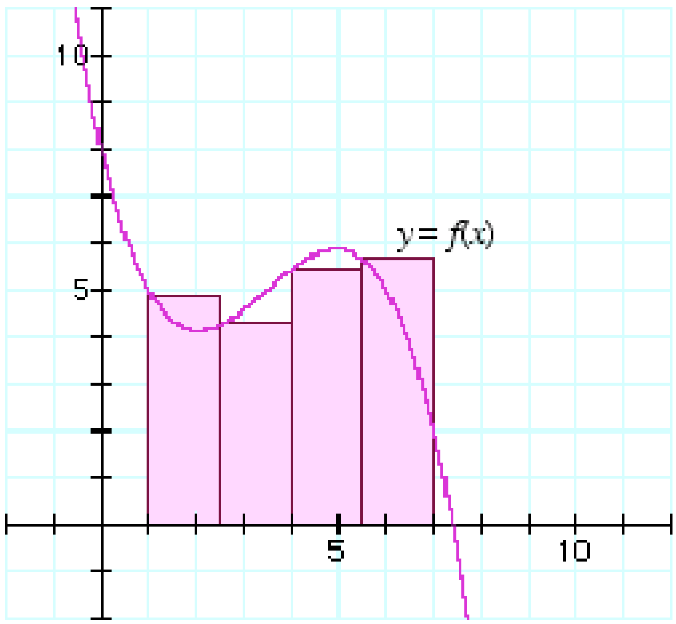
3.(a) $\int\frac{3x^2-8x+1}{x^3-4x^2+x+6}dx=\ln(|x-2|)+\ln(|x+1|)+\ln(|x-3|)+C.$

3.(b) $\int\frac{10x+2}{x^3-5x^2+x-5}dx=2\cdot\ln(|x-5|)-\ln(x^2+1)+C.$

3.(c) $\int\frac{x^4+3x^3+2x^2+1}{x^2+3x+2}dx=\frac{x^3}{3}+\ln(|x+1|)-\ln(|x+2|)+C.$

3.(d) $\int\frac{e^x}{(e^x-1)(e^x+2)}dx=\frac{1}{3}\ln(|e^x-1|)-\frac{1}{3}\ln(|e^x+2|)+C.$

4.(a) One possible area corresponding to $f(1)\cdot\frac{3}{2}+f\left(\frac{5}{2}\right)\cdot\frac{3}{2}+f\left(\frac{8}{2}\right)\cdot\frac{3}{2}+f\left(\frac{11}{2}\right)\cdot\frac{3}{2}$ is shown below. This sketch is based on the assumption that the sum is a left-hand Riemann sum. Other answers are possible (for example, by assuming that the sum is a right-hand Riemann sum or a midpoint sum).



4.(b) A Riemann sum that corresponds to $f(1)\cdot\frac{3}{2}+f\left(\frac{5}{2}\right)\cdot\frac{3}{2}+f\left(\frac{8}{2}\right)\cdot\frac{3}{2}+f\left(\frac{11}{2}\right)\cdot\frac{3}{2}$ is:
"The left-hand Riemann sum that approximates the area under $y=f(x)$ between $a=1$ and $b=7$." (Other answers are also possible here.)

4.(c) $\sum_{k=0}^{3} f\left(1+k\cdot\frac{3}{2}\right)\cdot\frac{3}{2}=30.25$.

4.(d) $\sum_{k=1}^{4} g(1+k\cdot\frac{3}{2})\cdot\frac{3}{2} < \sum_{k=0}^{3} g(1+\frac{3}{4}+k\cdot\frac{3}{2})\cdot\frac{3}{2} < \int_{1}^{7} g(x)dx < \frac{\sum_{k=0}^{3} g(1+k\cdot\frac{3}{2})\cdot\frac{3}{2}+\sum_{k=1}^{4} g(1+k\cdot\frac{3}{2})\cdot\frac{3}{2}}{2} <$
$\sum_{k=0}^{3} g(1+k\cdot\frac{3}{2})\cdot\frac{3}{2}$.

5.(a) $\int \frac{x^2}{\sqrt{9-x^2}}dx=\frac{-x\cdot\sqrt{9-x^2}}{2}+\frac{9}{2}\arcsin\left(\frac{x}{3}\right)+C$.

5.(b) $\int \frac{1}{t^2\cdot\sqrt{1+t^2}}dt=\frac{-\sqrt{1+t^2}}{t}+C$.

5.(c) $\int \frac{\sqrt{9-x^2}}{x^2}dx=\frac{-\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)+C$.

5.(d) $\int x^3 \cdot \sqrt{9-x^2} \cdot dx = \frac{1}{5}\left(9-x^2\right)^{\frac{5}{2}} - 3\cdot\left(9-x^2\right)^{\frac{3}{2}} + C$.

6.(a) Does not converge.

6.(b) Does not converge.

6.(c) Converges.

6.(d) Converges.

6.(e) More information needed.

6.(f) More information needed (specifically, what the limit of $x \cdot q(x)$ as $x \to \infty$.

7.(a) Converges to $\frac{\pi}{2}$.

7.(b) Diverges.

7.(c) Diverges.

7.(d) Converges to $\frac{1}{2} \cdot \ln\left(\frac{5}{3}\right)$.

8.(a) Trapezoid = 0.895122
Midpoint = 0.895478
Simpson = 0.898014

8.(b) Trapezoid = 9.649753
Midpoint = 9.650912
Simpson = 9.650526

8.(c) Trapezoid = 0.372299
Midpoint = 0.380894
Simpson = 0.376330

9.(a) The equation of the tangent line is $y = x + 1$.

9.(b) Since the graph of $y = e^x$ is concave up, the tangent line always lies at or below the curve. Therefore, the y-value produced by the tangent line from (a) (which is $1 + x$) must be less than or equal to the y-value of the curve (which is e^x).

9.(c) Substitute $1/x$ for x in the inequality from Part (b) and subtract 1 from both sides.

9.(d) Compare to the improper integral $\int_1^{\infty} \frac{1}{x^4} dx$ which, as a p-integral with $p > 1$, converges.

10.(a) $\int_3^5 x \cdot \cos(x) \cdot dx = \left[\cos(x) + x \cdot \sin(x)\right]_3^5 = -3.944$.

10.(b) $\int_1^3 t \cdot \ln(t) \cdot dt = \frac{9}{2}\ln(3) - 2.$

10.(c) $\int_0^5 \ln(1+t) \cdot dt = 6 \cdot \ln(6) - 5.$

10.(d) $\int_0^1 u \cdot \sin^{-1}(u^2) \cdot du = \frac{\pi}{4} - \frac{1}{2}.$