Recitation Handout 17: Radius and Interval of Convergence

Interval of Convergence

The interval of convergence of a power series: $\sum_{n=0}^{\infty} c_n \cdot (x-a)^n$ is the interval of *x*-values that can be plugged into the power series to give a convergent series.

The center of the interval of convergence is always the anchor point of the power series, *a*.

Radius of Convergence

The radius of convergence is half of the length of the interval of convergence. If the radius of convergence is *R* then the interval of convergence will include the open interval:

$$(a - R, a + R).$$

Finding the Radius of Convergence

To find the radius of convergence, *R*, you use the Ratio Test.

Step 1: Let $a_n = c_n \cdot (x-a)^n$ and $a_{n+1} = c_{n+1} \cdot (x-a)^{n+1}$.

Step 2: Simplify the ratio $\frac{a_{n+1}}{a_n} = \frac{c_{n+1} \cdot (x-a)^{n+1}}{c_n \cdot (x-a)^n} = \frac{c_{n+1}}{c_n} \cdot (x-a)$.

Step 3: Compute the limit of the absolute value of this ratio as $n \to \infty$.

Step 4: Interpret the result using the table below.

Limit of absolute value of ratio as $n \to \infty$.	Radius of convergence, *R*.
Zero.	Infinite. The power series converges for all values of *x*.
$N \cdot \lvert x-a \rvert$, where *N* is a finite, positive number.	$R = \frac{1}{N}$. The interval of convergence includes $\left(a - \frac{1}{N}, a + \frac{1}{N}\right)$ and possibly the end-points $x = a - \frac{1}{N}$ and $x = a + \frac{1}{N}$.
Infinity.	Zero. The power series converges at $x = a$ and nowhere else.

Are the end-points in the Interval of Convergence?

Each of the two end-points ($x = a - R$ and $x = a + R$) may or may not be part of the interval of convergence. To determine whether the end-points are in the interval of convergence, you have to plug them into the power series (one at a time) to get an infinite series. You then use a convergence test to determine whether or not the infinite series converges or diverges. If the infinite series converges, then the end-point that you plugged into the power series is in the interval of convergence. Otherwise, the end-point is not in the interval of convergence.

If you use the ratio test at each end-point you usually get an inconclusive test so it is best to try a different convergence test when investigating the end-points of the interval of convergence.

(a) Find the radius of convergence and interval of convergence for: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \cdot (x-1)^n$

$a_n =$

$\frac{a_{n+1}}{a_n} =$

Limit of absolute value of ratio =

RADIUS OF CONVERGENCE: ____________________

Find the radius of convergence and interval of convergence for: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \cdot (x-1)^n$

End points: and

Convergence or divergence at first end-point:

Convergence or divergence at second end-point:

INTERVAL OF CONVERGENCE: ______________________________

(b) Find the radius of convergence and interval of convergence for: $\sum_{n=0}^{\infty} 2^{2n} \cdot x^{2n}$

$a_n =$

$\frac{a_{n+1}}{a_n} =$

Limit of absolute value of ratio =

RADIUS OF CONVERGENCE: ____

Find the radius of convergence and interval of convergence for: $\sum_{n=0}^{\infty} 2^{2n} \cdot x^{2n}$

End points: and

Convergence or divergence at first end-point:

Convergence or divergence at second end-point:

INTERVAL OF CONVERGENCE: ______________________________

(c) Find the radius of convergence and interval of convergence for: $\sum_{n=1}^{\infty} \frac{4^n}{n} \cdot (x-3)^n$

$a_n =$

$\frac{a_{n+1}}{a_n} =$

Limit of absolute value of ratio =

RADIUS OF CONVERGENCE: ______________________________

Find the radius of convergence and interval of convergence for: $\sum_{n=1}^{\infty} \frac{4^n}{n} \cdot (x-3)^n$

End points: and

Convergence or divergence at first end-point:

Convergence or divergence at second end-point:

INTERVAL OF CONVERGENCE: ______________________________

(d) Find the radius of convergence and interval of convergence for:

$$1+2\cdot(x+5)+\frac{4!}{(2!)^2}\cdot(x+5)^2+\frac{6!}{(3!)^2}\cdot(x+5)^3+\frac{8!}{(4!)^2}\cdot(x+5)^4+\ldots$$

$a_n =$

$\dfrac{a_{n+1}}{a_n} =$

Limit of absolute value of ratio =

RADIUS OF CONVERGENCE: ______________________________

Find the radius of convergence and interval of convergence for:

$$1+2\cdot(x+5)+\frac{4!}{(2!)^2}\cdot(x+5)^2+\frac{6!}{(3!)^2}\cdot(x+5)^3+\frac{8!}{(4!)^2}\cdot(x+5)^4+\ldots$$

End points: and

Convergence or divergence at first end-point:

Convergence or divergence at second end-point:

INTERVAL OF CONVERGENCE: ____

ANSWERS:

(a) Radius of convergence = 1. Interval of convergence is (0, 2].

(b) Radius of convergence = 0.5. Interval of convergence is (−0.5, 0.5).

(c) Radius of convergence = 0.25. Interval of convergence is [2.75, 3.25).

(d) Radius of convergence = 0.25. Interval of convergence is (−5.25, −4.75).