We also described how one might proceed if the above objective was secondary to a normal linear objective, i.e. we wish to solve

$$P\text{: minimize } \zeta = \max_{x_j > 0} d_j$$

$$\text{subject to } \underset{x \geq 0}{Ax = b}$$

$$\text{and } \quad c'x = z^*,$$

where z^* = minimum of $c'x$ subject to $Ax = b$ and $x \geq 0$. Without loss of generality, we can assume $d_1 \leq d_2 \leq \ldots \leq d_n$. We show that problem P is solved if when using the simplex algorithm we choose as the variable to enter the basis that variable of lowest index which has a negative reduced cost.

We note first that on termination we will have $c'x = z^*$ and let x^* be the solution found and let t be such that $x_t^* > 0$ and $x_j^* = 0$ for $j > t$. Now consider the last time x_t was the incoming non-basic variable and let $\hat{z}$ be the objective value at this time. Then $\hat{z} > z^*$ as $x_t^* > 0$ and we can show that $\hat{z}$ is a lower bound to objective values obtained if we insist that $x_j = 0$ for $j \geq t$. Indeed, as the reduced costs are non-negative for $j = 1, \ldots t-1$ we have that $\hat{z}$ is the minimum objective value if we only allow $x_j > 0$ for $j \in \{1, \ldots t-1\} \cup \{i | x_i \text{ is currently basic}\}$. This completes the proof and we note that it is independent of the row selection rule used to avoid cycling (if any).

A. M. Frieze

REFERENCE

[1] A. M. Frieze (1975) Bottleneck linear programming. *Opl Res. Q.* **26**, 871–874.

A note on "A Study of the use of cloakrooms in office buildings" by P. J. Davidson and R. G. Courtney (1976) *Opl Res. Q.* **27**, 789–800.

We three kings (one actually of the orient) wish to comment with the best of humorous intentions upon the hygiene of males in office buildings.

Assuming that no male made use of wash-basin facilities *only*, Table 1 seems to suggest that 74% did not wash their hands after making use of the cloakroom facilities. But then (in the interests of hygiene) would the effect of a notice saying "Now wash your hands", mean the expenditure of providing more wash-basins?

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TABLE 2. RELATIVE PERCENTAGE ERRORS INCURRED IF FORMULAE (1) AND (5) ARE USED FOR THE EVALUATION OF THE AVERAGE QUEUEING TIME IN THE PROCESS $M/E_l/3$

ρ	Use of \ l	1	2	3	4	∞
0.5	(1)	0	−3.7	−5.4	−6.4	−9.5
	(5)	0	−0.5	−0.8	−0.9	−0.6
0.6	(1)	0	−2.7	−4.0	−4.7	−6.7
	(5)	0	−0.6	−0.8	−1.0	−0.6
0.7	(1)	0	−1.9	−2.7	−3.2	−4.4
	(5)	0	−0.5	−0.7	−0.8	−0.4
0.8	(1)	0	−1.1	−1.7	−2.0	−2.6
	(5)	0	−0.3	−0.5	−0.5	−0.3
0.9	(1)	0	−0.5	−0.8	−0.9	−1.2
	(5)	0	−0.2	−0.2	−0.3	−0.1

where $v = 1/l$. Formula (1) is also discussed by Maaløe[3] and can be derived, as a special case, from the more general approximate formulae presented by Rosenshine and Chandra.[4]

Tables 1 and 2 give the relative percentage errors incurred if formulae (1), above, and (5), established in reference[1], are used for the approximate evaluation of the average queueing time in some $M/E_l/r$ systems.

GEORGE P. COSMETATOS

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[2] F. S. HILLIER and F. D. LO (1972) *Tables for multiple-server queueing systems involving Erlang distributions*, Technical Report No. 149, Departments of Operations Research and Statistics, Stanford University, Stanford, California.

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[4] M. ROSENSHINE and M. J. CHANDRA (1975) Approximate solutions for some two-stage tandem queues, Part 1: Individual arrivals at the second stage. *Ops Res.* **23**, 1155–1166.

ON BOTTLENECK LINEAR PROGRAMMING

We have considered the problem[1]

$$\text{minimize} \quad \zeta = \max_{x_j > 0} d_j$$

$$\text{subject to} \quad \underset{x \geq 0.}{Ax = b}$$

and proposed an adaptation of the phase 1 simplex method for solving it.