Homework 3: due September 25

In all of these questions, the graph in question is $G_{n,p}$.

1. Let $k \geq 3$ be fixed and let $p = \frac{c}{n}$. Show that there exists $\theta = \theta(c,k) > 0$ such that w.h.p. all vertex sets S with $|S| \leq \theta n$ contain fewer than $k|S|/2$ edges. Deduce that w.h.p. either the k-core of $G_{n,p}$ is empty or it has size at least θn.

2. Let m_1^* be the hitting time for minimum degree 1 in the graph process. Suppose that $e_{m_1^*} = \{u, v\}$ where v is the vertex whose only incident edge is $e_{m_1^*}$. Show that w.h.p. there is no triangle containing u.

3. Let $G_{n,n,p}$ be the random bipartite graph with vertex bi-partition $V = (A, B)$, $A = [1, n]$, $B = [n+1, 2n]$ in which each of the n^2 possible edges appears independently with probability p. Let $p = \frac{\log n + \omega}{n}$, where $\omega \to \infty$. Show that w.h.p. $G_{n,n,p}$ is connected.