Homework 2: due September 11

1. Suppose that $0 < p < 1$ is constant. Show that w.h.p. $G_{n,p}$ has diameter two.

2. Let $f : [n] \to [n]$ be chosen uniformly at random from all n^n functions from $[n] \to [n]$. Let $X = \{j : \nexists i \; s.t. \; f(i) = j\}$. Show that w.h.p. $|X| \approx e^{-1}n$.

3. Suppose that $p = \frac{c}{n}$ where $c > 1$ is a constant. Show that w.h.p. the giant component of $G_{n,p}$ is non-planar. (Hint: Argue that the number of edges in the giant is asymptotically equal to $(c+x)/2$ times the number of vertices in the giant. You can assume that $c + x > 2$. Remove vertices from the giant so that the girth is large. Now use Euler's formula for the case when the graph has large girth. Note also that an ν-vertex planar graph of girth g has at most $\nu(1 - 2/g)^{-1}$ edges.)