Homework 1: due September 4

In all of these questions, the graph in question is $G_{n,p}$.

1. Suppose that $p = \frac{\log n}{n}$. Let $S = \left\{v : degree(v) \leq \frac{\log n}{20}\right\}$. Prove that w.h.p. S contains no edges.
2. Suppose that $p = \frac{c}{n}$ where c is a constant and that $s_0 = n/(e^2c^2)$. Show that w.h.p. all sets of vertices of size $s \leq s_0$ contain at most $2s$ edges.
3. Suppose that $p = \frac{c}{n}$ where c is a constant. Show that w.h.p. there are no two cycles of size at most 10 that share a vertex.