HW4: 3.3.1; 4.3.3 4.3.7.

Hints and information:

3.3.1 Just compute the expected number of vertices of degree k in $G_{n,m}$ and show it is asymptotic to $\frac{d^k e^{-d}}{k!}n$.

You will need to calculate the number of graphs with vertex set $[n]$, m edges and where vertex 1 has k neighbors. Then multiply this by n and divide by $\binom{N}{m}$. Then carefully simplify the expression you get.

This is not enough to solve the problem as stated, but enough forthe homework.

4.3.3 Let $\mathcal{A}_i, i = 0, \dots, i_0 = \left\lfloor \frac{2 \log n}{3 \log d} \right\rfloor$ be the event that $|S_i(v)| \in [(d/2)^i, (2d)^i]$ for all $v \in [n]$. Use induction and the Chernoff bounds to show that $\mathbf{P}(\neg \mathcal{A}_{i+1} \mid S_i(v), \mathcal{A}_j, j \leq i) \leq n^{-\omega}$ where $\omega \to \infty$.

4.3.7 The number of spanning trees in the complete bipartite graph $K_{r,s}$ is $r^{s-1}s^{r-1}$. If $G_{n,n,p} = (X, Y, E)$ is not connected then there exists $K \subseteq X, L \subseteq Y$ such that $s = |K| + |L| \leq n$. Somewhere in your calculations you should find that $2k\ell \leq (k + \ell)^2/4$ comes in handy.