

Homework 7

7.6.10 Show that if $d > 2k \log k$ for a positive integer $k \geq 2$ then w.h.p. $G(n, d/n)$ is not k -colorable.

(Hint: Consider the expected number of proper k -coloring's:

$$\sum_{n_1 + \dots + n_k = n} \binom{n!}{n_1! \dots n_k!} (1-p)^{\sum_i n_i(n_i-1)/2}.$$

7.6.11 Let $p = K \log n/n$ for some large constant $K > 0$. Show that w.h.p. the diameter of $\mathbb{G}_{n,p}$ is $\Theta(\log n / \log \log n)$.

(Breadth First Search.)

11.6.10 Show that w.h.p. $\mathbb{G}_{n,3}$ is not planar.

(Remove short cycles and consider Euler's formula when there are no small faces.)