# Department of Mathematical Sciences Carnegie Mellon University <br> 21-393 Operations Research II <br> Test 2 

Name: $\qquad$

| Problem | Points | Score |
| :--- | :--- | :--- |
| 1 | 33 |  |
| 2 | 33 |  |
| 3 | 34 |  |
| Total | 100 |  |

## Q1: (33pts)

Find a minimum weight path from $s$ to all other nodes of the digraph below. The 2 numbers $a, b$ on the directed edge $v w$ are to be interpreted as follows: If a path $P$ from $s$ to $v$ has weight $t$, then the weight of the path $P+v w$ is $t+(a+b t)$.


Q2: (33pts) Find the optimal ordering strategy for the following inventory system. If you order an amount $Q$, it costs $A Q^{1 / 2}$ and the inventory cost is $I$ per unit per period. The demand is $\lambda$ units per period and no stock-outs are allowed.

Q3: (34pts) Two players A,B simultaneously choose an integer between 1 and 5 . If the numbers are equal there is no payoff. The player that chooses a number one larger than than that chosen by their opponent wins two. The player that chooses a number two or more larger than than that chosen by their opponent loses one.
(a) Set up the payoff matrix for this game.
(b) What is the value of this game?
(c) Argue that there is an optimal solution in which the strategy for A satisfies $p_{1}=p_{5}$ and $p_{2}=p_{4}$. Find such a strategy and a strategy for B.
(d) Show that what you have found is a stable solution.

