# Department of Mathematical Sciences Carnegie Mellon University <br> 21-393 Operations Research II <br> Test 2 

Name: $\qquad$

| Problem | Points | Score |
| :--- | :--- | :--- |
| 1 | 25 |  |
| 2 | 30 |  |
| 3 | 20 |  |
| 4 | 25 |  |
| Total | 100 |  |

Q1: (25pts)
(a) Write down the dual of

$$
\begin{array}{ll}
\operatorname{maximise} & x_{1}+6 x_{2} \\
\text { subject to } & \\
& 2 x_{1}+3 x_{2} \leq 12 \\
& 6 x_{1}+x_{2} \leq 8 \\
& x_{1}, x_{2} \geq 0
\end{array}
$$

(b) You are now given that the optimal solution to the above program is $x_{1}=0, x_{2}=4$. Use complementary slackness to solve the dual.

Q2: (30pts)
Write down the Karush-Kuhn-Tucker conditions for the following problem:

$$
\text { minimise }(x-1)^{2}+(y-2)^{2}
$$

subject to

$$
2 x+3 y_{2} \geq 10
$$

Solve the problem by finding a solution to the KKT conditions.

Q3: (20pts)
Set up the initial tableau for solving the problem of Q2 by the restricted simplex algorithm. List the pairs of variables that cannot simultaneously be basic. -YOU DO NOT HAVE TO CONTINUE BEYOND THIS POINT IN THE SOLUTION OF THE PROBLEM

## Q4: (25pts)

Players A and B play the following game. A chooses a number
$x_{A} \in\{0,1,2,3\}$ and $B$ chooses a number $x_{B} \in\{0,1,2\}$. If $x_{A}+x_{B}$ is odd, A wins a point, otherwise B wins a point.
Write down a linear program whose solution will produce an optimum strategy for A. YOU DO NOT HAVE TO SOLVE THE PROGRAM.

