**OPP: Optimal Pricing Project**

***What’s more OP than OP?...OPP***

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**Background and Motivation**

Today’s entertainment industry is primarily composed of three major forms, music, movies, and sports. In order to attend any of the events above, potential customers must purchase a ticket for admission. The biggest player in the ticket sales market is Ticketmaster, which has over 9,000 exclusive agreements, giving them control over 90% of the United State’s arenas and amphitheaters, 70% of the clubs and small theaters, and most of the basketball, hockey, and football games. This has led them to garner the nickname, the modern monopoly of the ticket market.

Although Ticketmaster has no bearing on the prices of the tickets they sell, the primary goal of any ticketing service is to sell as many tickets as possible at the highest price. The system that is currently being employed by Ticketmaster has content owners price the tickets followed by Ticketmaster listing them for sale at that same price from the day it is released to the day of the event. According to this method, CEO of Ticketmaster, Nathan Hubbard has admitted that roughly 40% of their tickets will go unsold.

The most significant issue stemming from this current model is determining the listing price. Here, this listing price is the only price the public will be offered for as long as they are for sale. While there are millions of dollars spent every year trying to understand consumer preferences and market demand, there are too many variables such as time, place, and the type of event, just to name a few, to always find that optimal price which will maximize revenue.

Another example of this model is in the film industry. Movie theaters set ticket prices and once they are set, all movies screened at that theater would cost that price. This model does not take into account for newly released films, which receive the highest demand for their opening released showings. Furthermore, films with higher budgets such as superhero movies tend to gross more than low budget independently financed movies. However, currently both tickets are priced equal which again does not take into account the relationship between ticket price and demand. Therefore, in order to maximize revenue, one would expect theaters to set their prices in way, which relates the sale price of their tickets to the demand for which they generate.

This static pricing strategy as well as the rise in the Internet marketplace has led to a rapid increase in the secondary ticket market. StubHub, the largest secondary ticket site allows scalpers to list tickets on their website for whatever price they wish. For high demand events such as a Justin Bieber concert or a Dallas Cowboys game, scalpers will buy up all the tickets on Ticketmaster upon their release and then post them on StubHub for a price, which takes into account the elevated market demand. This difference is money lost out by content providers such as the Biebster for employing this current system.

The basis of this pricing strategy model is using market research to anticipate what the demand will be for a particular event as opposed to a linear programming model. This will consider different demand functions, which will allow the price of tickets to adjust according to the fluctuations demonstrated within our demand function.

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**Modeling**

We will construct a model that optimizes our ticket pricing based on consumer demand. There are many different demand functions. As we noted earlier, high demanded events such as a New York Yankee game or a Justin Beiber concert has demand that is not met by capacity. Conversely, a Pirates Game or Carnegie Mellon football game has more capacity than demand. Our model aims to maximize revenue in both these scenarios.

The model currently used by Ticketmaster uses a flat rate independent of time or demand level. We believe that we can improve on this model by incorporating variables and parameters ignored by the current model. In order to include the complexity of ticket prices and the flexibility of being able to use a various number of demand functions, we will subsequently model demand as a non-linear exponential model. Through this we will obtain a constrained optimization problem that can be solved using non linear programming, in particular KKT conditions.

Our model uses multiple demand functions that are dependent on ticket price, a point in time that is prior to the actual event, and parameters that allow us simulate varying demand over time. We believe that demand is not constant over time. This is reinforced by ticket sales that fluctuate in the time before an event. For instance, there is often an influx of demand right before a popular event or even when tickets are first available for purchase. Sometimes when tickets are first available, there is not that much demand, but as time increases, demand also increases as the event approaches. Examples include sporting events such as ice skating and tennis. There are even instances where demand decreases over time independently of price. We thus chose a form of the demand function with parameters beta and alpha that allow us to model demand over time.

**Problem:**

The model problem we will attempt to solve is:

With d(pi)= alpha\*e^-beta\*p

Alpha= the scaling to make demand interpretable with respect to actual tickets sold and capacity. Alpha >=0

Beta= constant that is adjustable according to demand level. Basically beta will be low when there is high demand and high when there is low demand. 0<beta<1

Pi= the price of a ticket on a given period, i

Di(pi) is the demand on period i, given the price on period i.

**Assumptions:**

-Demand is known for each period i for each possible price that is set.

- Market data suggests that we model the demands as we have done above

- Demand is influenced by time

- Each demand period is independent

**Demand Functions:**

We will explore many different demand functions over time assuming a constant price level. Specifically we will explore 4 different demand situations as listed below:

**Case1: Monotonic Decreasing**

The demand is high initially but decreases over time. A real world example would be the demand for an event that lacks public interest. We will look into a specific example, namely the Pittsburgh Pirates. For season passes, the demand will initially be relatively high due to loyal fans of Pittsburgh, who are unaffected by poor team performance. However, as these true fans diminish, demand will ultimately converge to 0.

The parameters we are using are alpha and beta. As i increases over time, we will model our parameter beta such that beta is decreasing. So beta may start at .25 but increase to a value of 1 as demand decreases. Alpha will stay constant as a scaling factor. Demand and beta are inversely related.

**Case 2: Monotonic Increasing**

The demand is initially low, but then increases over time. We suspect that this is a rare and very specific event in the real world. Such an event would not be popular initially, but demand would increase over time due to media hype and word of mouth. There would be no traditional advertising for this type of an event.

The parameters we are using are alpha and beta. As i increases over time, we will model our parameter beta such that beta is decreasing. So beta may start at 1 but decrease to a value of .25 as demand increases. Alpha will stay constant as a scaling factor.

**Case 3: Concave**

The demand increases to a maximum and then decreases over time at a constant price. A real world example of this would be demand increasing for an event as a result of marketing. So, there would not be high demand initially but as marketing increases demand increases to a maximum. However, the effect of marketing will diminish over time, so demand will ultimately decrease to initial demand levels. Subsequently, our parameters will reflect this demand behavior over time.

The parameter beta will initially decrease as demand increases but as demand hits a maximum, beta will hit a minimum, and increase as demand decreases. Alpha will stay constant as a pricing factor.

**Case 4: Convex**

The demand is high initially, then tapers off, and then increases as it nears the event. A real world example of this would be any realistic popular entertainment event. When the tickets first go on the market many diehard fans will pick up the tickets immediately. Thus there will be a high demand. However, generally the demand of true fans decreases because many of them have already bought the tickets early. Following the initial burst in demand, there might be a slight lull in demand due to no new customer base. However as the event nears, demand will start increasing again due to last second purchases of the event and possible word of mouth marketing.

The parameter beta will initially increase as demand decreases but as demand hits a minimum, beta will hit a maximum value, and decrease as demand increases. Alpha will stay constant as a pricing factor. We can calculate alpha using the equation: alpha =demand/e^-1. We will partition beta(i) into nodes that equal the capacity, and depending on the demand function, assign node values to each beta(i). Since beta is inversely related to demand, we will use beta =1-(demand/capacity)/c.

**Sample Problem: Capacity is Greater than Aggregate Demand**

The Tartans football team is trying to sell tickets and wants to know what they can charge for a ticket over a three day period. If price was constant, the base demand for day 1, 2 and 3 would be 40, 20, and 10, respectively. We are given that the capacity for the event in 100 seats and people will pay $1 for a ticket even if they don’t want to go.

We determine that gamma is 1. We determine that alpha (1) = 40/e^-1= 108.73. We will use beta= .60, b=.80, and beta=.90. We will now calculate the optimal price at each level which is gamma/beta. So on day 1, 2 and 3, we should charge $1.67, 1.25, and 1.10 respectively. Since capacity is greater than demand, the constraint function will be naturally satisfied. Q.e.d.

**Solving the Problem using Karush Kuhn Tucker Conditions:**

A potential problem with the solution provided about is that capacity is usually less than aggregate demand over a period of time. Firms must decide how to allocate their ticket sales effectively by selling more tickets at higher prices and avoid selling tickets at lower prices. In order to solve our problem we use Karush Kuhn Tucker conditions.

We must solve the following problem:

D(i) is the forecasted demand on day i.

**Sample Problem using a Two Period Model:**

KKT Conditions:

Since we assume prices are greater than 0, we will consider the following cases:

**Case 1:**

: then , which suggests that but since , and , and . Thus, when , capacity equals the sum of aggregate demand, and thus we will allocate so that our demand functions equal the forecasted demand.

**Case 2:**

If , , but since we assume that p1 and p2 is greater than 0, then . Solving for p2 and p1, we find that p1=1/, and p2 =1/. This was the optimal solution that we found.

**Case 3:**

This follows the same logic as the first case.

**In summary, if does not equal 0, then p can be maximized by equating the forecasted demand and the demand function and solving. If , then we follow the optimal solution that we discovered earlier.**

Discrete time steps vs. continuous?

1. Simplicity- it would be impossible to represent this as a dynamic programming problem.
2. Realistic Application- it would not be feasible for ticket firms to continuously modify ticket prices every second. Thus, we choose a period in which ticket prices could be changed. Similarly, we cannot measure demand continuously over time. Most statistical data is measured in a period of time.

**Optimality:**

The current model is only optimal as demand stays constant over time. However, in current economics, this is not a feasible assumption. Thus if the demand shifts left or right as shown, then the price level is no longer optimal. Our pricing model, adjusts for these changes in demand, so we can always set the optimal price level.

Within the restrictions and assumptions of our model, we do have an optimal solution to pricing. However, realistically we cannot assume that the ticket sellers are in full knowledge of demand over a period of time. However there are several ways to forecast demand using statistical or operations strategy. These may be added measures to ensure that our model is optimized realistically. However, we do believe that under the scope our information given to us, our model satisfies optimality. Shown above, we are able to choose the optimal price as the demand function shifts. This gives us confidence that our model is optimal.

**Further applications**

The linear programming theory underlying our pricing algorithm has many more applications beyond ticket pricing for entertainment events. For instance, we can generalize the ticket idea to any event that has a finite supply with a changing demand function. For instance, airline flights have a limited number of seats available because of constraints imposed by each plane. Demand for each of these seats can follow any number of demand functions, based on the destination (more popular destinations will have initially higher demand that may wane over time; more mundane destinations may have concave demand functions), and it is the goal of the airline to maximize the revenue earned from selling tickets to willing consumers. Likewise, movies have a limited number of seats in each theater and demand changes over time based on factors like marketing and word-of-mouth reviews. Sporting events follow the same pattern, and as such ticket pricing for athletics is another application of our model.

After broadening the scope of potential applications, we find that our model can be put towards pricing finite but renewable items. Perishable food has decreasing demand as time progresses as the value of freshness is priced into the demand function. Utility decreases inversely with time, but with renewing batches replacing expired items, we find that this is a closely-related but more complex application of our model. Similarly, hotel rooms are finite but supply is renewed continuously as tenants leave. The prevalence of last-minute demand and early reservations imply that hotel booking’s most appropriate demand function is concave. Hotel reservations are also linked to related time-varying models of airline ticket pricing in that booking a flight reservation usually correlates to booking a hotel room, as well.

Finally, there are applications to portfolio optimization as linear programming is able to model the time-varying demand for certain asset classes based on economic conditions and personal risk preferences, able to incorporate capital budgeting as a constraint to act analog to the finite supply of tickets in our original model. In general, linear programming is used extensively in microeconomics and other operations research topics, which include planning, transportation and production involving commoditized events and items.