Pittsburgh Public Schools Assignment

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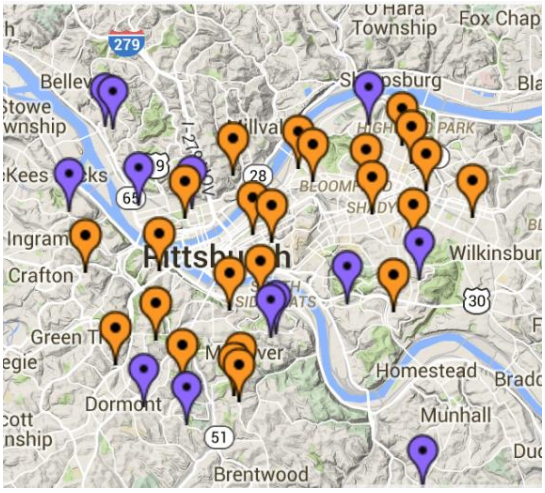
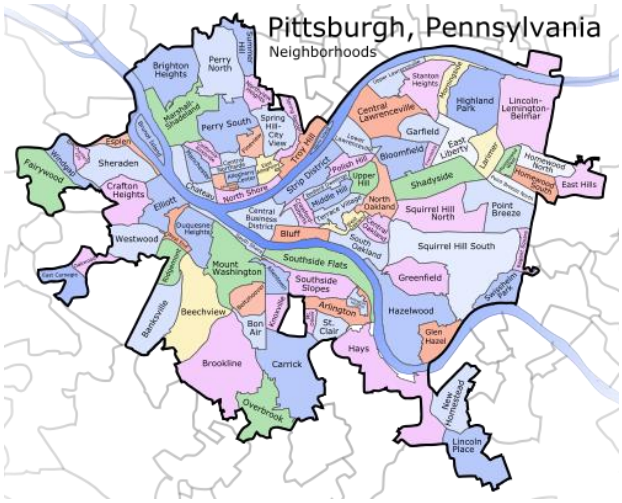
Introduction

Public schools are crucial players in the development of youth, providing equal access to educational opportunities and preparing them to be responsible citizens. 90% of school students in the United States attend public schools. There are approximately 3.1 million teachers and 49.8 million students in public schools as of fall 2014; giving an average student-to-teacher ratio of 16.1. Countless studies have shown that students learn more effectively in smaller classes.

Barriers to education exist in the form of transportation. Attendance is one of the most important factors determining a child's academic achievement. Poor school performance and higher dropout rates are correlated with absenteeism. Getting to school is difficult when students do not have reliable transportation options. In addition, if a child is not eligible for school-provided transportation and has to walk to school, the route to school may be hazardous, especially for younger children.

Pittsburgh Public Schools, the public school district of Pittsburgh, is the largest school district in Allegheny County and the second largest in the state of Pennsylvania. Currently 54 schools serve approximately 25,000 students in Kindergarten through Grade 12. The average student-teacher ratio for academic year 2011-2012 was 17.4. Elementary school students who live more than 1.5 miles from their schools are provided with bus transportation from the Transportation Department.





Left: Map of Pittsburgh K-5 (orange) and K-8 (purple) public schools
Right: Map of Pittsburgh neighborhoods

In this report, we seek to assign students to schools that minimize the distance students are traveling to get to school to address some of these issues. Without information on individual students in Pittsburgh, we use the aggregation of students by residential neighborhoods to assign neighborhoods to schools rather than individuals. The approach we take assigns proportions of each neighborhood to schools. To start, we minimize the total distance between the neighborhoods and schools. As the neighborhoods have different populations, we also examine the total distance between the

neighborhoods and schools with distances weighted by the number of students living in that neighborhood. The last model examines the maximum distance between any neighborhood and school.

Data

The University Center for Social and Urban Research at the University of Pittsburgh compiled neighborhood demographics from the 2010 Census. We gathered data on the residential population for ages 5-9 in the 39 Pittsburgh neighborhoods between the Allegheny River and Monongahela River from the Center's 2011 research report.

The National Center for Education Statistics (NCES) located within the U.S. Department of Education is the primary federal entity for collecting and analyzing education-related data. Using the Elementary/Secondary Information System web application, basic data were obtained on full-time equivalent teacher counts, pupil-teacher ratios, and addresses for K-5 and K-8 schools in the Pittsburgh school district for the academic year 2011-2012. The 17 schools between the Allegheny River and Monongahela River were of interest.

Distances between the neighborhoods and schools were calculated using the Google Maps Application Programming Interface.

Assumptions

Public schools provide tuition-free education that all students are eligible for. The problem we were interested in solving involves assuming all kids from the neighborhoods go to public schools. Although statistics show that 10% of school students nationally attend private schools, we wanted to assign every child to a public school since this is an education option families have. As these children between the two rivers reside within the city limits, we also assumed they would only be eligible to enroll in schools in the Pittsburgh school district. Additionally, we assumed kids living between the two rivers would only enroll in the schools between the two rivers.

We were only interested in assigning students who attend Kindergarten to 5th grade. At Pittsburgh Public Schools, a child must turn 5 years old by September 30th of the target school year to be eligible to register for Kindergarten. Then by the beginning of 5th grade, a child will be 10 years old. As the population numbers we collected were for children aged 5-9, we increased those counts by 5% to account for this.

There was no information on the geographic distribution of the children in the neighborhoods. Another assumption we made was that the kids in each neighborhood all reside in the center of the neighborhood. This simplification averages the distance effects for each neighborhood.

We assumed there was no maximum capacity for the number of students and the number of teachers at any school. Additional portable classrooms were considered to be accessible to the schools if needed. A maximum student-teacher ratio of 16 is decided on to mirror the national average.

Lastly, we did not look to force any race or gender quotas at these schools. Achieving racial diversity and gender equality was not considered to be important in our model. It seems reasonable to assume neighborhoods are gender-balanced.

Initial Method

To begin the experiment, we tried to model the problem as a variation of set covering that tried to minimize the distance given the location of the schools and the neighborhoods. We allow the neighborhoods to be broken up into sections, assigning fractions of a neighborhood to different schools. We take the total distance into account. Without taking any other factors into account we set up the problem as follows with the following variable definitions. The results of this method are presented after the model.

n : number of neighborhoods $= 39$
s : number of schools $= 17$
x_{ij} : proportion of neighborhood i going to school j
d_{ij} : distance between neighborhood i and school j
p_i : population of neighborhood i

Unweighted First run

$$\text{minimize} \sum_{i=1}^{n}\sum_{j=1}^{s} d_{ij} * x_{ij}$$

subject to:

$$\sum_{j=1}^{s} x_{ij} = 1 \text{ each neighborhood is only assigned to one school}$$

$$\sum_{i=1}^{n} \frac{x_{ij}p_i}{t_j} \leq 16 \text{ each school has a student:teacher ratio of less than 16}$$

$$0 \leq x_{ij} \leq 1$$

Total Distance (mi)		95.52		
Minimum Total Neighborhood distance	Crawford	0.25		
Maximum Total Neighborhood distance	Squirrel Hill South	8.01		
Largest Population Distance			**Smallest Population Distance**	
Squirrel Hill South		8.01	West Oakland	1.06
Squirrel Hill North		5.01	Central Business District	0.35
Highland Park		6.77	Strip District	6.79
Hazelwood		3.72	Central Oakland	3.73
Garfield		4.47	Bluff	0.43
Mean		5.596	**Mean**	2.472

The overall total distance traveled in this solution is 95.52 miles. When observing the results, we see that the neighborhood that has to travel the farthest is Squirrel Hill South at 8.01 miles. It is also the most populated neighborhood. When comparing the five largest neighborhoods to the five smallest, the mean distance traveled in the larger neighborhoods is 3.124 miles greater than in the smaller neighborhoods. In context of the problem, this does not seem optimal. The larger neighborhoods, with the majority of the population, have to travel a longer distance. This would complicate transportation and seem to add unnecessary movement to such a large population. Since the algorithm did not take into account the size of the population, it was only trying to optimize the shorter distance. As the amount of transporting is a key factor in the problem, we looked at other methods to incorporate the variables and optimize different parts.

Alternative Algorithms

When dealing with facility location problems, there is very rarely one optimal solution. Depending on the situation, there are many methods with different objective functions and goals. When looking at the situation and the overall objective, it is then important to decide which method provides the best solution given the real world context. The methods we researched and tested are P-center and P-median.

P-Median

The P-Median algorithm focuses on minimizing the total weighted distance. In this particular scenario it involves minimizing the total distance of travel weighted by the population of each neighborhood. This is similar to the first method as described above except with the additional parameter of population. The advantages of this algorithm is that it will place more emphasis on the neighborhoods with larger weights, making sure they get placed at the closest schools possible. By doing so, the overall solution has the most kids being as close as possible, which could be advantageous. The negatives of the p-median is that it can result in some small neighborhood being allocated to a school very far away due to its minimal influence in the overall city. When dealing with a situation where kids need to get to school at some reasonable time, especially relative to their peers, this could be a problematic situation. The formulation and results are presented below. This also involves the same variable definitions as presented in the first method.

P-Median

$$\text{minimize} \sum_{i=1}^{n} \sum_{j=1}^{s} d_{ij} * x_{ij} * p_i$$

subject to:

$$\sum_{j=1}^{s} x_{ij} = 1 \text{ each neighborhood is only assigned to one school}$$

$$\sum_{i=1}^{n} \frac{x_{ij} p_i}{t_j} \leq 16 \text{ each school has a student:teacher ratio of less than 16}$$

$$0 \leq x_{ij} \leq 1$$

Total Distance (mi)		126.6		
Minimum Total Neighborhood distance	Squirrel Hill South	0.187		
Maximum Total Neighborhood distance	Homewood North	8.35		
Largest Population Distance			**Smallest Population Distance**	
Squirrel Hill South		0.187	West Oakland	1.37
Squirrel Hill North		1.95	Central Business District	1.87
Highland Park		6.77	Strip District	1.37
Hazelwood		3.73	Central Oakland	4.09
Garfield		4.47	Bluff	2.24
Mean		3.4214	**Mean**	2.188

The overall total distance traveled in this solution is 126.6 miles. This distance is 31.08 miles more than in the previous algorithm, showing the difference in objectives between P-median and the first method. The farthest any neighborhood has to travel is 0.24 miles more than before, but that neighborhood, Homewood North, is an average sized neighborhood. The key difference in this method is that the top five neighborhoods travel an average of 1.17 miles less than in the first method. With 688 residents residing in Squirrel Hill South, this is much larger than the average neighborhood size of 155. For the algorithm to assign Squirrel Hill South to closer schools than before, it significantly reduces the amount of transportation needed compared to the past problem. However for a neighborhood to have to travel around 8.35 miles to school is still far when others can travel much less. The next method looks to address this issue.

P-Center

The P-center algorithm's objective is to minimize the maximum distance of the solution. It not only takes the population of the neighborhood into account, but also works to minimize how far the farthest distance is, even if that means rearranging some of the shorter distances. The benefits of the method is that neighborhoods with smaller populations will not be totally neglected. Also since it tries to minimize the maximum distance, it can provide a solution that has less variation between the shortest and longest distance. When looking for an implementation that supports equality throughout the city, this can provide a viable and fair solution. The disadvantage is that it can push those close to schools farther away to make the overall maximum distance smaller. Similar to before, the setup and results are presented below, with the same variable definitions.

P-Center

$$\text{minimize } w \text{ where } w = \max\{d_{ij} * x_{ij}\}$$
$$\text{subject to:}$$

$$\sum_{j=1}^{s} x_{ij} = 1 \text{ each neighborhood is only assigned to one school}$$

$$\sum_{i=1}^{n} \frac{x_{ij} p_i}{t_j} \leq 16 \text{ each school has a student:teacher ratio of less than 16}$$

$$0 \leq x_{ij} \leq 1$$

Total Distance (mi)		175.23	
Minimum Total Neighborhood distance	Squirrel Hill North	1.96	
Maximum Total Neighborhood distance	Lower Lawrenceville	4.99	
Largest Population Distance		**Smallest Population Distance**	
Squirrel Hill South	4.43	West Oakland	4.48
Squirrel Hill North	1.96	Central Business District	4.32
Highland Park	4.79	Strip District	4.26
Hazelwood	4.04	Central Oakland	4.68
Garfield	4.25	Bluff	4.01
Mean	3.894	**Mean**	4.35

The overall total distance traveled in this solution is 175.23 miles. The P-center algorithm proved effective in its objective as it was able to reduce the maximum distance traveled by any neighborhood from 8.01 miles from the first method to 4.99 miles. Overall the total distance traveled in the city is significantly larger than in the previous methods. In addition both the five most-populated and least-populated neighborhoods experience a rise in their mean distance traveled compared to the P-median method. However for the larger neighborhoods, the additional distance is small, and for the smaller areas, the mean distance increased by 2.16 miles. The overall range of how far all the neighborhoods have to travel is smaller than in both previous methods, and provides a more equal level of travel no matter how large the population is. When dealing with school systems, these results provide a more equal distribution of distances, and ensures that no neighborhood is traveling over 8 miles like in the previous methods.

Conclusion

School enrollment decisions in the community are affected by transportation circumstances. Our assignment problem sought to optimize the distance students in Pittsburgh neighborhoods would travel to get to the Pittsburgh public schools they attend. In our solution we explored three different methods: basic set-covering, P-median, and P-center. Each method has its own advantages and

disadvantages in terms of interpretability and practicality. The methods in increasing order of the overall total distance traveled for all neighborhoods is: basic set-covering, P-median, and then P-center. When we didn't include neighborhood populations into the basic set-covering model, some of the most populated neighborhood ended up having the longest distances. This influenced us to pursue the two additional approaches. The P-median algorithm gets the most kids as close as possible to their schools. The P-center algorithm provided a more uniform solution across neighborhoods and ignored neighborhood population sizes. Which method is best will ultimately depend on the priorities of the school district. Additionally, it may be worth finding integer programming solutions to these three models for simplicity.

Changes could be made to the data prior to modeling to simplify the problem. We could combine neighborhoods that are adjacent to increase the stability of smaller-sized neighborhoods and to minimize the number of variables in our problem. Similarly, we could group neighboring schools together for the assignment.

Due to the large number of variables in our models, we decided to keep the constraints to a minimum. There are some other constraints worth exploring as additions to the models. With a greater number of neighborhoods (39) than schools (17), we could introduce the constraint that no more than k neighborhoods be assigned to given school for some integer k to be determined. Also, we could ensure that each school has at least one neighborhood assigned to it if we assume school closures are not an option. Lastly, achieving racial diversity and gender equality may be of an area of interest: for 2011-2012, Pittsburgh Faison K-5 had 1 American Indian/Alaskan, 3 Asian/Pacific Islander, 4 Hispanic, 6 White, 14 Mixed, and 685 Black students enrolled.

A different problem involves taking the student-teacher ratio constraint and attempting to minimize it instead. A constraint in this situation could be setting an upper bound on the distance traveled. The other possible constraints mentioned previously could also apply here.

Sources

- http://nces.ed.gov/
- http://www.pps.k12.pa.us/
- http://ucsur.pitt.edu/wp-content/uploads/2014/11/UCSUR_SF1_NeighborhoodProfiles_July2011.pdf
- http://www.centerforpubliceducation.org/Main-Menu/Public-education/An-American-imperative-Public-education-/default.aspx
- http://www.nassp.org/tabid/3788/default.aspx?topic=A_Focus_on_Attendance_Is_Key_to_Success
- http://nepc.colorado.edu/files/pb_-_class_size.pdf

APPENDIX

Appendix A: First Method

	ARSENAL K-5	COLFAX K-8	DILWORTH K-5	FAISON K-5	FORT PITT K-5	FULTON K-5	GREENFIELD K-8	LIBERTY K-5	LINCOLN K-5	LINDEN K-5	MIFFLIN K-8	MILLER K-5	MINADEO K-5	MONTESSORI K-8	SUNNYSIDE K-8	WEIL K-8	WOOLSLAIR K-5
Central Oakland	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Shadyside	0.138888889	0	0	0	0	0	0.781746032	0.079365079	0	0	0	0	0	0	0	0	0
Friendship	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
South Oakland	0	0	0	0	0	0	0	0	0	0	0	0	0.78028169	0	0	0	0.21971831
Bloomfield	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Bluff	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
North Oakland	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Point Breeze North	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Lower Lawrenceville	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Greenfield	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
Strip District	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Terrace Village	0	0	0	0	0	0	0	0	0	0	0	0.603864734	0.273429952	0	0	0.122705314	0
Squirrel Hill North	0	0	0.458468677	0.109048724	0	0.090487239	0	0	0	0	0	0	0	0.34199536	0	0	0
West Oakland	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Squirrel Hill South	0	0	0.340697674	0	0	0	0	0	0	0.531395349	0.127906977	0	0	0	0	0	0
Central Business District	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Highland Park	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
East Liberty	0.803636364	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.196363636
Ridgemont	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Polish Hill	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Morningside	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Upper Lawrenceville	0	0	0	0.064516129	0	0	0	0	0	0	0	0	0	0	0	0	0.935483871
Swisshelm Park	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crawford-Roberts	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Central Northside	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Regent Square	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Point Breeze	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Garfield	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Hazelwood	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Central Lawrenceville	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Larimer	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Bedford Dwellings	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Homewood North	0	0.093726937	0	0	0	0	0.538745387	0	0	0	0	0	0	0	0.367527675	0	0
Middle Hill	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
East Hills	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Upper Hill	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Homewood West	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Homewood South	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Glen Hazel	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix B: P-Median

	ARSENAL K-5	COLFAX K-8	DILWORTH K-5	FAISON K-5	FORT PITT K-5	FULTON K-5	GREENFIELD K-8	LIBERTY K-5	LINCOLN K-5	LINDEN K-5	MIFFLIN K-8	MILLER K-5	MINADEO K-5	MONTESSORI K-8	SUNNYSIDE K-8	WEIL K-8	WOOLSLAIR K-5
Central Oakland	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Shadyside	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Friendship	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
South Oakland	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bloomfield	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
Bluff	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
North Oakland	0	0	0	0	0	0.866666667	0	0	0	0	0	0.133333333	0	0	0	0	0
Point Breeze North	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Lower Lawrenceville	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Greenfield	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
Strip District	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Terrace Village	0	0	0	0.333333333	0	0	0	0	0	0	0	0.471497585	0	0.195169082	0	0	0
Squirrel Hill North	0	0	0.182830626	0	0	0	0.817169374	0	0	0	0	0	0	0	0	0	0
West Oakland	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Squirrel Hill South	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Central Business District	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Highland Park	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
East Liberty	0.376727273	0	0	0	0	0	0	0.054545455	0	0	0	0	0.083636364	0	0	0.485090909	0
Ridgemont	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Polish Hill	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Morningside	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Upper Lawrenceville	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
Swisshelm Park	0	0	0	0	0	0	0	0	0	0	0.653846154	0	0	0	0.346153846	0	0
Crawford-Roberts	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Central Northside	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Regent Square	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Point Breeze	0.249668874	0.18013245	0.570198675	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Garfield	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Hazelwood	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Central Lawrenceville	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Larimer	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Bedford Dwellings	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Homewood North	0	0	0	0	0	0	0	0	0.894464945	0	0	0	0	0	0	0.105535055	0
Middle Hill	0	0	0	0	0	0	0	0	0	0	0	0	0.8	0.2	0	0	0
East Hills	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Upper Hill	0	0	0	0	0	0	0	0	0	0	0	0.432653061	0	0	0	0	0.567346939
Homewood West	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Homewood South	0	0	0	0	0	0	0.311731844	0	0.688268156	0	0	0	0	0	0	0	0
Glen Hazel	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0

Appendix C: P-Center

	ARSENAL K-5	COLFAX K-8	DILWORTH K-5	FAISON K-5	FORT PITT K-5	FULTON K-5	GREENFIELD K-8	LIBERTY K-5	LINCOLN K-5	LINDEN K-5	MIFFLIN K-8	MILLER K-5	MINADEO K-5	MONTESSORI K-8	SUNNYSIDE K-8	WEIL K-8	WOOLSLAIR K-5
Central Oakland	0.055284117	0.071665675	0.059595544	0.060800535	0.05641416	0.058366196	0.055303387	0.055219071	0.058787972	0.059207203	0.059299553	0.057238767	0.060130184	0.058568657	0.059059778	0.058055523	0.057003676
Shadyside	0.02901518	0.004231127	0.078499852	0.003026136	0	0.063275229	0.029257977	0.029521438	0	0.040435268	0.072781616	0.050329172	0.349622677	0.067417251	0.07280987	0.062078724	0.047698482
Friendship	0.049974999	0.120742356	0.060753566	0.063766044	0.022986645	0.057680195	0.050023173	0.049812384	0.058734636	0.059782714	0.060013589	0.054861623	0.062090166	0.058186349	0.059414151	0.056903514	0.054273895
South Oakland	0.060548402	0	0.260490009	0	0	0.071497616	0.060616809	0.060311496	0	0.074466138	0.074771392	0.067504464	0	0.072230185	0.060487807	0.070401183	0.0666745
Bloomfield	0.040712661	0.006043892	0.082609537	0.002421733	0	0.069623258	0.040908248	0.040526944	0.049887331	0.076939347	0.078215627	0.059433891	0.177327555	0.073135636	0.076459763	0.068503504	0.057251072
Bluff	0.053514411	0.078086748	0.059981551	0.061789038	0.055209475	0.058137529	0.053543316	0.053416842	0.058770194	0.05939904	0.059537565	0.056446385	0.060783512	0.058441221	0.059177902	0.05767152	0.056093749
North Oakland	0.050859852	0.087718358	0.060560562	0.063271792	0.053402449	0.057794528	0.050903209	0.050713499	0.058743526	0.059686796	0.059894583	0.055257814	0.061763503	0.058250067	0.059355089	0.057095516	0.054728858
Point Breeze North	0.069512956	0.010845995	0.117547075	0.003143395	3.50314E-07	0.072404399	0.069587144	0.18968921	0.016645856	3.50314E-07	0.074176902	0.072095846	0.000946421	0.075525066	0.081968193	0.076410954	0.069499886
Lower Lawrenceville	0.057197462	0	0.191158462	0	0	0.071527586	0.057287066	0.056895095	0.06584414	0.075437343	0.075866065	0.066286229	0	0.072470637	0.07475175	0.070084693	0.065193472
Greenfield	0.015674089	0.079188791	0.075535149	0.005857552	5.89912E-05	0.058753942	0.015960243	0.014968384	3.52787E-05	0.083992085	0.072180596	0.043919954	0.310399819	0.061587015	0.067492282	0.056048789	0.038347037
Strip District	0.055992	0.082650786	0.059441141	0.060405134	0.056896034	0.058457662	0.056007415	0.055939963	0.045241543	0.059130469	0.059204349	0.057555719	0.059868853	0.058619632	0.059012528	0.058209125	0.057367646
Terrace Village	0.03922556	0.00655937	0.083848826	0	0.050921503	0.07112507	0.039425001	0.038552335	0.075490459	0.079829502	0.080785324	0.059456183	0.098324068	0.073220549	0.078303648	0.067909613	0.057022989
Squirrel Hill North	0	0.519723636	0.053523427	0.130150701	0	0.027030968	0	0	0.036120256	0.045154688	0.04714483	0.002734879	0.065044924	0.03139402	0.041977671	0	0
West Oakland	0.057230794	0.064602495	0.059170936	0.059713182	0.057739313	0.058617729	0.057239465	0.057201523	0.058807529	0.058996183	0.05903774	0.058110386	0.059411524	0.058708837	0.058929841	0.058477927	0.058004595
Squirrel Hill South	0	0.395904923	0.116728244	0.016255498	0	0	0	0	0	0.226929579	0.234541022	0	0	0	0.008823916	0	0.000816817
Central Business District	0.056345941	0.067813032	0.05936394	0.060207433	0.057136971	0.058503396	0.05635943	0.056300409	0.058798639	0.059092101	0.059156746	0.057714196	0.059738188	0.058645119	0.058988903	0.058285925	0.057549632
Highland Park	0.186169994	0.056455566	5.16806E-07	0.118816523	2.38419E-08	0.001742608	0.161456249	0.221064181	8.27306E-07	2.38419E-08	0.001211389	0.000477158	0.076310134	0.050310712	8.32725E-07	0.101729495	0.024253767
East Liberty	0.029410647	0.013380832	0.071997465	0.00544326	4.2736E-07	0.051464511	0.029675604	0.040449239	8.7691E-07	9.18248E-07	0.000247645	0.044914918	0.472594714	0.064652677	0.069953822	0.062678263	0.043134179
Ridgemont	0.055995733	0	0.216879419	0.060413983	0.056892992	0.058467203	0.056011149	0.055941899	0	0.059139318	0.059201169	0.057563324	0.059877702	0.058628481	0.029395512	0.058215623	0.057376495
Polish Hill	0.050505911	0.112206526	0.060637764	0.063469493	0.053161512	0.057748795	0.050551195	0.050353053	0.035536017	0.059725163	0.059942186	0.055099337	0.061894168	0.05822458	0.059378714	0.057018715	0.054546873
Morningside	0.082938969	0.018432993	0.008415913	0.033979825	6.12393E-07	0	2.78506E-05	0.391731605	6.12393E-07	6.7409E-05	2.4289E-05	0.078561636	0.029671473	0.090407492	0.089060941	0.0980865	0.07859188
Upper Lawrenceville	0.073218361	0.002610829	0.071107589	0.011012606	5.73953E-07	0.070162577	0.073307965	0.218118659	5.73953E-07	0.069602205	0.010367133	0.075352502	0.008359117	0.085759159	0.068171501	0.084366654	0.078481994
Swisshelm Park	0.074613976	0.018684999	3.85403E-05	0.006212615	3.0723E-07	0	0.43396708	0.074531451	5.01018E-07	3.0723E-07	0.007326893	0.076075561	0.003987109	0.059603911	0.092590981	0.079432066	0.072933702
Crawford-Roberts	0.054747859	0	0.129603615	0	0.060906592	0.071545185	0.054852878	0.054393359	0.073843868	0.076128678	0.076631985	0.065400698	0	0.072648602	0.075325209	0.069852021	0.064119451
Central Northside	0.054441008	0	0.214221357	0	0.051058576	0.071546803	0.054547954	0.054079977	0	0.076214318	0.076725035	0.065290486	0	0.072671692	0.075393473	0.069823276	0.063986043
Regent Square	0.051030937	0.282405177	0.060499594	0.063154107	0	0.057774044	0.05107333	0.050887365	0.014431272	0	0.022240215	0.055321277	0.061679334	0.058243975	0.05933998	0.057118377	0.054801016
Point Breeze	0.036895482	0.140816061	0.057836621	0.304757272	0	0.029386578	0.037186453	0.065863617	0.022572898	7.39147E-07	0.04941241	0.041363923	0	0.052217513	0.079407198	0.053696946	0.02858629
Garfield	0	0.00022486	0.066683679	0.548127696	0	0.047442996	0	0	0	0	0.055041697	0.029802013	0.075053126	0.050615231	0.058298941	0.042586495	0.026123267
Hazelwood	0	0.000234099	0.065328356	0.438030263	0.011025773	0.045167041	0	0	0.052084179	0.058959571	0.060474111	0.02667721	0.074096455	0.048487415	0.056541794	0.040072018	0.022821715
Central Lawrenceville	0.063366201	0.010398269	0.159930162	0.014337408	7.46135E-07	0.067200676	0.000464913	0.152963381	9.18595E-07	0.068649885	0.065869448	0.071085114	0.009772267	0.086123284	0.073042606	0.083191291	0.07360343
Larimer	0.070592292	0.013454948	0.00219326	0.006997895	4.8278E-07	0.070937132	0.070705019	0.217829253	0.015695216	0.059138013	0.074186611	0.072521134	0.003659635	0.081406325	0.085384714	0.083046887	0.072251182
Bedford Dwellings	0.058420676	0	0.116794431	0	0.063223358	0.071519508	0.058502572	0.058144231	0.07331206	0.075093792	0.075486279	0.066727936	0	0.072379971	0.074467234	0.070199152	0.065728799
Homewood North	0.05843684	0.056243979	8.30405E-05	0.012665228	4.64811E-07	4.64811E-07	0.232235999	0.066916372	0.028839736	4.64811E-07	0.000209272	0.055279479	0.19555235	0.066278818	0.108036892	0.074147283	0.045073316
Middle Hill	0.060716187	0	0.10952374	0	0.064671337	0.071503461	0.060783631	0.060488526	0.07224463	0.074446988	0.074770213	0.06755746	0	0.072212077	0.073930999	0.070416108	0.066734641
East Hills	0.062357654	0.070756469	2.85008E-06	0.01549969	9.76953E-07	8.81325E-06	0.426871863	0.06912345	0.07193462	0	0	0.064085431	0.00202158	0.000139864	0.133617561	0.083198481	0.000380695
Upper Hill	0.056431233	0	0.165846286	0	0.061968443	0.071533416	0.056525654	0.056112508	0.031486575	0.075654355	0.07610687	0.066009016	0	0.072525479	0.074931971	0.070011123	0.064857069
Homewood West	0.050680102	0.222013028	0.060578038	0.063355077	0	0.057749703	0.050724422	0.050533832	0.038247812	0	0.059890589	0.055165233	0.06181327	0.058221758	0.059358284	0.057046553	0.0546223
Homewood South	0.041883328	0.321825683	0.071598519	0	0	0	0.042055791	0.041301984	0.07323704	0	0.07781576	0.059374853	0	0.071276487	0.07567515	0.066685505	0.057269901
Glen Hazel	0.052270048	0.188100572	0.060236701	0.062472534	0	0.057965293	0.052305697	0.05214992	0.058735268	0.008512651	0.059700255	0.055885325	0.061232385	0.05834356	0.059245121	0.057396325	0.055448344

Appendix D: Google Maps API

```
<!DOCTYPE html>
<html>
 <head>
  <title>Distance Matrix service</title>
  <script src="https://maps.googleapis.com/maps/api/js?v=3.exp"></script>
  <style>
   html, body {
    height: 100%;
    margin: 0;
    padding: 0;
   }

   #map-canvas {
    height: 100%;
    width: 50%;
   }
   #content-pane {
    float:right;
    width:48%;
    padding-left: 2%;
   }
   #outputDiv {
    font-size: 11px;
   }
  </style>
  <script>
var map;
```

```
var geocoder;
var bounds = new google.maps.LatLngBounds();

function initialize() {
 var opts = {
  center: new google.maps.LatLng(55.53, 9.4),
  zoom: 10
 };
 map = new google.maps.Map(document.getElementById('map-canvas'), opts);
 geocoder = new google.maps.Geocoder();
}

function calculateDistances() {
 var service = new google.maps.DistanceMatrixService();
 service.getDistanceMatrix(
  {
   origins: ["Squirrel Hill, Pittsburgh, PA", "Shadyside, Pittsburgh, PA"]
   destinations: ["Arsenal Elementary School, Pittsburgh, PA","Beechwood Elementary School,
Pittsburgh, PA"],
   travelMode: google.maps.TravelMode.DRIVING,
   unitSystem: google.maps.UnitSystem.METRIC,
   avoidHighways: false,
   avoidTolls: false
  }, callback);
}

function callback(response, status) {
 if (status != google.maps.DistanceMatrixStatus.OK) {
  alert('Error was: ' + status);
 } else {
  var origins = response.originAddresses;
  var destinations = response.destinationAddresses;
  var outputDiv = document.getElementById('outputDiv');
  outputDiv.innerHTML = '';
  deleteOverlays();

  for (var i = 0; i < origins.length; i++) {
   var results = response.rows[i].elements;
   addMarker(origins[i], false);
   for (var j = 0; j < results.length; j++) {
    addMarker(destinations[j], true);
    outputDiv.innerHTML += origins[i] + ' to ' + destinations[j]
       + ': ' + results[j].distance.text + ' in '
       + results[j].duration.text + '<br>';
   }
  }
 }
}
```

```
function addMarker(location, isDestination) {
 var icon;
 if (isDestination) {
  icon = destinationIcon;
 } else {
  icon = originIcon;
 }
 geocoder.geocode({'address': location}, function(results, status) {
  if (status == google.maps.GeocoderStatus.OK) {
   bounds.extend(results[0].geometry.location);
   map.fitBounds(bounds);
   var marker = new google.maps.Marker({
    map: map,
    position: results[0].geometry.location,
    icon: icon
   });
   markersArray.push(marker);
  } else {
   alert('Geocode was not successful for the following reason: '
    + status);
  }
 });
}
function deleteOverlays() {
 for (var i = 0; i < markersArray.length; i++) {
  markersArray[i].setMap(null);
 }
 markersArray = [];
}

google.maps.event.addDomListener(window, 'load', initialize);
  </script>
 </head>
 <body>
  <div id="content-pane">
   <div id="inputs">
    <pre>
    </pre>
    <p><button type="button" onclick="calculateDistances();">Calculate
     distances</button></p>
   </div>
   <div id="outputDiv"></div>
  </div>
  <div id="map-canvas"></div>
 </body>
</html>
```