Optimizing Jungle Paths in League of Legends

21-393 Final Project

Fall 2013

Taylor Caligaris Isa Daher Andrew Kharma William Veer

Introduction

League of Legends (LOL) is a popular videogame internationally. It is classified as a multiplayer online battle arena game (MOBA).During these games two teams composed of five players each compete to complete objectives and ultimately reach the other team's base and destroy their nexus. Each player chooses an avatar of sorts called a champion. Each champion has different strengths, weaknesses and abilities. These champions can be further customized by the use of runes and masteries which grant bonuses to a variety of a champion's statistics. There are five major roles within each team: top lane, mid lane, attack damage carry (AD carry), support, and jungler.

In the past year it has experienced greatly growing popularity with the introduction of League of Legends Championship Series (LCS) for both the North America and European servers. In each LCS the game producers choose the top eight professional teams in each region to compete weekly and receive a salary for the given season. These weekly competitions are streamed live on both the game producers' website and on YouTube. The game has received such popularity that the world finals for 2013 took place in the LA Lakers stadium, in order to accommodate the number of fans interested in attending. With the growing recognition of professional League of Legends players, many of their techniques and strategies have become commonplace in casual play. We are interested in testing the optimality of these techniques for the role of jungler.

Problem

The beginning portion of the game is called the "laning phase." During this phase it is the responsibility for the various players in lanes to attempt to take the enemy's towers in their respective lanes and to progress up the lanes towards the enemy's base and nexus. For these champions who have given lanes they are nearly always in conflict with the enemy. During the laning phase it is the jungler's role to traverse the team's jungle and kill the various neutral monsters that generate there. A map of the game "arena" is shown below in Appendix D1. Killing these monsters allows the jungler to accumulate the experience required to level up and the gold required to purchase new items containing bonus statistics. Also the absence of the jungler from a lane means that he is not depriving his teammates from any gold or experience associated with laning.

There are 5 major camps in each team's jungle. Each contains different monsters with different statistics. Each of the monsters at the camps will attack the jungler once he is in range and attempt to kill the jungler until they themselves are dead. The monsters regenerate after a fixed interval of time, allowing the jungler to clear a camp many times over. The statistics of the monsters also change over time based on how long it has been since they most recently were generated. For example the Ancient Golem at the Blue camp spawns with 1400 health at 1:55 into the game; by 2:55 if he hasn't been killed his health will be up to 1490. All of the statistics associated with the monsters is contained in Appendix A.

We are interested in finding the ideal path between camps through the jungle to minimize the time for the jungler to reach level 6. We decided to choose Warwick as our champion because a Warwick player does not go into lane before level 6. Warwick's ultimate ability allows for a very easy time killing an enemy champion in lane. Without this ability, he does not have any kill potential.

Assumptions

We ignore the dragon as it is a very team oriented objective that isn't thought of until later on. We also ignore Baron as it does not spawn until the 15 minute mark.

We automatically use skills when they are available and when there are enemies to use them on.

The spell, Flash (a short range teleport), will not be factored into our calculations as it has no effect on the jungling part of the game.

As stated above, Warwick does not want to "gank" lanes until he is level 6. Therefore we are ignoring lanes in our calculations.

All decisions are made instantaneously. All skills are used perfectly. The "player" knows whether or not he can survive a camp before doing it.

We also assumed that our starting point would be the Ancient Golem camp. There are only 2 possible starting camps: Ancient Golem and Elder Lizard (they spawn a good 15 seconds before the other camps) and Elder Lizard grants a buff that is only useful for ganking while the Ancient Golem buff is extremely useful for sustaining oneself in the jungle.

Proposed Method

We decided to use a greedy algorithm to find an 'optimal' path as finding the actual optimal path would require an enormous number of calculations. The reason for this complexity is that a camp does not always provide the same amount of reward. The rewards and costs of a camp are constantly changing based on the player's level, the items the player has, the time since the last spawning of a camp, etc. Each individual path has it's own reward to cost ratio even though the player is going to the same camps.

Since the goal was to reach level 6 as soon as possible, we ignored gold gain and focused entirely on experience gain. Starting at the Ancient Golem camp, we found the ratios of experience gained to total time spent for each other camp. We decided to go to the camp with the highest ratio. We repeated this until we had reached 2500 experience, the required amount for level 6. If at any point, our health was too low to do another camp, we returned to base. We also returned to base if we were low on mana (resource used to cast abilities).

Results

Camp	Travel Time	Battle Time	Experience Earned	Cumulative Gold Earned	Current Gold	Cumulative Experience Earned	Notes
							Level 2 Reached. Blue
Blue	0	18	360	105	105	360	Buff Obtained.
Wolves	6	19	190	199	199	550	
							Level 3 Reached. Red
Red	24	23	363	322	322	913	Buff Obtained.
Golems	10	18	246	438	438	1159	Level 4 Reached.
							Blue Buff Gone. Out of
							Mana. Back to Base +
Wolves	32	11	193	529	529	1352	10 sec. Buy Madred's.
Golems	44	11	240	633	233	1592	Red Buff Gone.
Wolves	32	14	192	719	319	1784	Level 5 Reached.
Golems	32	10	239	821	421	2023	
Blue	25	14	361	929	529	2384	Blue Buff Obtained.
Wolves	6	14	192	1014	614	2576	Level 6 Reached.

This took 6 minutes and 13 seconds.

Camp	Monster	Quantity	Initial Spawn	Respawn Timer	Gold	Experience	Health
	Wraith	1			30	103	1000
Wraiths			2:05	0:50	(+0.72/min)	(+2.1/min)	(+39/min)
	Lesser Wraith	3		0.00	3	4	150
					(+0.054/min)	(+0.16/min)	(+18/min)
	Giant Wolf	1			55	170	1100
Walvas		Ŧ	2.05	0.50	(+0.65/min)	(+2.3/min)	(+40/min)
woives	Wolf	2	2:05	0.50	4	10	200
	VV OII	2			(+0.081/min)	(+0.32/min)	(+25/min)
	Dia Calam	1			55	160	1200
Calana	Big Golem	T	2.05	0:50	(+0.8/min)	(+2.4/min)	(+48/min)
Golems	C. L	4	2:05		15	38	300
	Golem	T			(+0.23/min)	(+1.6/min)	(+37/min)
		_			66	260	1400
Blue	Ancient Golem	1	1:55	5:00	(+0.332/min)	(+2.5/min)	(+90/min)
	Young Lizard	2			5	50	400
		4			66	260	1400
Red	Lizard Elder	1	1:55	5:00	(+0.342/min)	(+2.5/min)	(+90/min)
	Young Lizard	2			5	50	400
_				6.00	190 (+25 for	100	3500
Dragon	Dragon	1	2:30	6:00	killer)	400	(+240/min)

Appendix A: Jungle Monster Statistics

Camp	Monster	Attack Damage	Attack Speed	Critical Strike Chance	Armor	Magic Resist	Movement Speed
W/raiths	Wraith	35 (+0.75/min)	0.638/sec	0%	15	0	330
vvraitiis	Lesser Wraith	10 (+0.33/min)	0.638/sec	0%	5	0	330
Walvas	Giant Wolf	35 (+0.7/min)	0.679/sec	25%	9	0	443
wolves	Wolf	8 (+0.43/min)	0.679/sec	25%	6	0	443
Coloma	Big Golem	54 (+1.6/min)	0.613/sec	0%	12	-10	350
Golenis	Golem	30 (+1.3/min)	0.613/sec	0%	12	-10	350
	Ancient	65					
Blue	Golem	(+1.825/min)	0.613/sec	0%	20	0	200
	Young Lizard	11	0.679/sec	0%	8	0	330
		65					
Red	Lizard Elder	(+1.825/min)	0.625/sec	0%	20	0	330
	Young Lizard	11	0.679/sec	0%	8	0	330
Dragon	Dragon	145	0.658/sec	0%	21	30	335

- Blessing of the Lizard Elder (a.k.a. "Red Buff"): Additional incentive to clear the "Red" camp. "Red Buff" attaches to the champion who kills the Lizard Elder at the Red camp for 2.5 minutes. This unit's physical attacks apply a debuff that slows the target's movement speed by 8%¹ (5%¹ for ranged attacks) for 3 seconds and a damage-over-time that deals 8+2·Level bonus true damage twice. If the buff holder is slain, this buff is transferred to the killer.
- Crest of the Ancient Golem (a.k.a. "Blue Buff"): Additional incentive to clear the "Blue" camp. "Blue Buff" attaches to the champion who kills the Ancient Golem at the Blue camp for 2.5 minutes. This unit regenerates 25 flat mana regeneration per 5 seconds and 0.5% of their maximum mana per second and has 20% cooldown reduction on their abilities. If slain, this buff transfers to the killer.

¹ Assuming champion level less than 6

Appendix B: Build for Warwick: The Blood Hunter²

B1: Base Statistics

Health	428 (+98)	Attack Damage	56.76 (+3.375)
Health Regeneration	7.05 (+0.8)	Attack Speed	0.679 (+2.88%)
Mana	190 (+30)	Armor	16 (+3.5)
Mana Regeneration	7.1 (+0.6)	Magic Resist	30 (+1.25)
Range	125	Movement Speed	345

B2: Runes

Name	Number	Total Effect	
Greater Quintessance of Hybrid Penetration	2	+5.37 Armor Penetration	
Greater Quintessence of Hybrid Penetration	5	+4.2 Magic Penetration	
Greater Mark of Hybrid Depatration	2	+2.7 Armor Penetration	
	5	+1.86 Magic Penetration	
Greater Mark of Attack Speed	6	+10.2% Attack Speed	
Greater Seal of Armor	9	+12.69 Armor	
Greater Glyph of Scaling Ability Power	9	+1.53 Ability Power per Level	

B3: Masteries

Offensive Masteries

Name	Description					
	Improves the following Summoner Spells:					
	Exhaust: reduces target's Magic Resist and Armor by 10					
Summoner's Wrath	Ignite: Increases Ability Power and Attack Damage by 5 while on cooldown					
	Ghost: Increases Movement Speed bonus to 35%					
	Garrison: Allied Garrisoned turrets deal 50% splash damage					
Fury	+3% Attack Speed					
Deadliness	+0.67 attack damage per level					
Weapon Expertise	Physical Damage ignores 8% of the target's armor					

Defensive Masteries

Name	Description					
	Improves the following Summoner Spells:					
	Cleanse: Increases duration of disable reduction by 1 second					
Summoner's Resolve	Heal: Passively increases Health by 5 per level					
	Smite: Grants 10 bonus gold on use					
	Barrier: Increase shield amount by 20					
Durability	+6 Health per level					
Tough Skin	Reduces damage taken from monsters by 2					

²Based on the build guide by saintvicious, season 3 jungler for Curse.

http://www.lolpro.com/guides/warwick/69-warwick-build-guide-jungle-saintvicious

Hardiness	+2 Armor
Bladed Armor	Deals 6 true damage to any minion or monster that attacks you
Unyielding	Reduces damage taken from champions by 2
Veteran's Scars	+30 Health
Block	Reduces damage taken from champion basic attacks by 3
Tenacious	Reduces the duration of crowd control effects by 15% (stacks multiplicatively with Tenacity)
Juggernaut	Increases maximum Health by 4%
Defender	Grants 1 Armor and Magic Resist for each nearby enemy champion
Honor Guard	Reduces damage taken by 3%

B4: Summoner's Spells

Name	Description	Base Cooldown
Flash	Teleports your champion to target nearby location under your mouse cursor	300
Smite	Deals 460+ (level·30) true damage to target enemy minion or pet	70

B5: Skills and Skill Order

Skill Order

Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Hungering Strike		х	х		х		х		х									
Hunters Call	х													х	х		х	х
Blood				×				x		x		×	x					
Scent				^				^		^		~	^					
Infinite						v					v					~		
Duress						^					^					^		

Skills³

Name	Description	Cooldown	Mana Cost	Bonus Statistics
Eternal Thirst	Passive: Each of Warwick's autoattacks deal 3/3.5/4/4.5/5/5.5/6/6.5/7/8/9/10 /11/12/13/14/15/16 additional magic damage and heal him for the same amount. Each successive attack against the same target will stack this amount of haling and damage up to a maximum of 3 stacks. The stacks remain for 4 seconds.	NA	NA	

³ Skill details taken from League of Legends wiki http://leagueoflegends.wikia.com/wiki/Warwick

Hungering Strike	Active: Warwick swipes at a target enemy, dealing magic damage for the greater value between a flat minimum amount or a percentage of the target's maximum health (if the target is a champion), plus additional bonus damage based on his ability power. Hungering Strike can only deal flat damage to minions and monsters. Warwick heals himself for 80% of the damage dealt (after reductions)	10/9/8/7/ 6	70/80 /90 /100 /110	MAX MAGIC DAMAGE TO CHAMPIONS: 8 / 10 / 12 / 14 / 16% of target's max health (+ 100% AP) FLAT MINIMUM DAMAGE: 75 / 125 / 175 / 225 / 275 (+ 100% AP)
Hunters Call	Active: For 10 seconds, Warwick increases his attack speed and increases the attack speeds of all allies within range by half as much.	24/22/20/ 18/16	35	ATTACK SPEED: 40 / 50 / 60 / 70 / 80% ALLY ATTACK SPEED: 20 / 25 / 30 / 35 / 40%
Blood Scent	Toggle: Warwick reveals enemy champions within range with less than 50% health. While any enemies are revealed this way, Warwick has increased movement speed.	4	NA	MOVEMENT SPEED: 20 / 25 / 30 / 35 / 40%
Infinite Duress	Active: Warwick blinks to the front of an enemy champion and channels for 1.5 seconds, suppressing the target for 1.8 seconds and dealing magic damage 5 times in 1/3-second intervals. Warwick gains 30% life steal for the duration. Each damaging strike applies on-hit effects, life steal, and maximum stacks of Eternal Thirst.	90/80/70	100/ 125/ 150	MAGIC DAMAGE PER HIT: 50 / 67 / 84 (+ 40% bonus AD) TOTAL DAMAGE: 250 / 335 / 420 (+ 200% bonus AD)

B6: Item Buy Order

Starting Items

Namo	Description		Builds
Name	Description	Cost	From*
Huptoric	Unique Passive-Maim: basic attacks deal 10 bonus magic damage to		
Machata	monsters on hit	300	NA
Machete	Unique Passive-Butcher: Damage dealt to monsters increased by 10%		
Health	Click to Consume: Restores 150 Health over 15 seconds.		NIA
Potionsx2			NA
Sight	Click to Consume: Places and invisible ward that reveals the		NIA
Ward	surrounding area for 3 minutes	75	NA

Full Game Build

Namo	Description		Builds
Name	Description	Cost	From*
Madred's Razors	+25 Armor Unique Passive-Maim: Basic attacks deal 60 bonus damage to monsters on hit	400	Hunter's Machete

Vampiric	+10 Attack Damage	800	ΝΔ
Scepter	+10% Life Steal	800	NA
	+25 Attack Damage		
Wriggle's Lantern	+25 Armor		Madred's
	+15% Life Steal		Razor
	Unique Passive-Maim: Basic attacks deal 100 bonus magic damage to	500	and
	monsters on hit		Vampiric
	Unique Active: Places an invisible Sight Ward that reveals the		Scepter
	surrounding area for 90 seconds (90 second cooldown)		

*new item description overwrites those of items from which it was built

Appendix C: Distances and Times between Camps

		Ending Camp						
		Base	Wolves	Blue	Wraiths	Red	Golems	Dragon
	Base	0	8344	8749	8622	9050	8724	11707
	Wolves		0	2146	3866	4803	6345	6878
Ctouting	Blue		2146	0	6041	9050	8749	9354
Camp	Wraiths		3866	6041	0	1846	3576	3489
Camp	Red		4803	9050	1846	0	1998	4778
	Golems		6345	8749	3576	1998	0	4829
	Dragon		6878	9354	3489	4778	4829	0

C1: Distances (Game Distance Units) between Camps

C2: Time (seconds) between Camps

Warwick's movement speed is 345, at all levels considered in this problem. Time between camps is Distance/Movement Speed. Time from any jungle camp to base is 8 seconds⁴.

		Ending Camp						
		Base	Wolves	Blue	Wraiths	Red	Golems	Dragon
	Base	0	24.2	25.4	25.0	26.2	25.3	33.9
	Wolves	8.0	0	6.2	11.2	13.9	18.4	19.9
Chanting	Blue	8.0	6.2	0	17.5	26.2	25.4	27.1
Camp	Wraiths	8.0	11.2	17.5	0	5.4	10.4	10.1
Camp	Red	8.0	13.9	26.2	5.4	0	5.8	13.8
	Golems	8.0	18.4	25.4	10.4	5.8	0	14.0
	Dragon	8.0	19.9	27.1	10.1	13.8	14.0	0

⁴ http://leagueoflegends.wikia.com/wiki/Recall

Appendix D: Miscellaneous Game Information

D1: Map of Summoner's Rift



D2: Definitions and Calculations of Various Game Statistics

	Number of autoattacks per second
	maximum attack speed is 2.5
Attack Speed	Bonus attack speed stacks additively (Example: Stinger grants +40% attack speed and Nashor's Tooth grants +50% attack speed, so the champion gains +90% in total.)
Critical Strike Chance	When averaged over a large number of autoattacks critical strike chance is essentially a damage multiplier where each % of critical strike chance adds 1% damage. Thus at 50% critical strike chance a champion will be doing (on average) 150% damage with autoattacks.
	Maximum critical strike chance is 100%
	Life steal is an offensive statistic that converts a percentage of the physical damage dealt to health regained.
Lifestear	Life steal is calculated from the actual damage done to an enemy, after armor and all other forms of damage reduction.
Armor/ Armor	When attacking, armor penetration and armor reduction are considered in the following order: (1) flat armor reduction, (2) percentage armor reduction, (3) percentage armor penetration, (4) flat armor penetration
Penetration/ Armor Reduction	Incoming physical damage is multiplied by a factor based on the unit's armor If armor≥0, damage multiplier=100/(100+armor)
	If armor<0, damage multiplier=2-[100/(100-armor)]
Movement Speed	The rate at which a champion travels across a map. One movement speed point translates to one game distance unit traveled per second.
Movement Speed	Movement Speed=(Base Movement Speed + Flat Movement Bonuses) × (1 + Percentage Movement Bonuses) × (Slow Ratio × Slow Resist Ratio)
Magia Desistante	Magic Resistance reduces the damage of incoming magical attacks by a percentage.
Magic Resistance	If magic resistance>0, multiplier=1-[Magic Resistance/(100+ Magic Resistance)] If magic resistance≤0, multiplier=2-[100/(100- Magic Resistance)]
Passive Gold Gain	16 gold is gained per 10 seconds beginning 90 seconds into the game

D3: Champion Experience per Level

To reach level	Experience	Cumulative Experience
2	280	280
3	390	670
4	500	1170
5	610	1780
6	720	2500

Appendix E: Warwick's Full and Relevant Statistics by Level and Item Set

E1: Starting Items

	Level 1	Level 2	Level 3	Level 4	Level 5		
Health	578	680	782	884	986		
Health Regeneration	7.85	8.65	9.45	10.25	11.05		
Mana	220	250	280	310	340		
Mana Regeneration	7.70	8.30	8.90	9.50	10.10		
Attack Damage	60.81	64.85	68.90	72.94	76.99		
Attack Speed	0.788	0.808	0.827	0.847	0.866		
Armor	34.2	37.7	41.2	44.7	48.2		
Magic Resist	31.3	32.5	33.8	35.0	36.3		
Movement Speed	345	345	345	345	345		
Range	125	125	125	125	125		
Armor Penetration	8.1	8.1	8.1	8.1	8.1		
Magic Penetration	6.1	6.1	6.1	6.1	6.1		
Ability Power	1.5	3.1	4.6	6.1	7.7		
Lifesteal	0	0	0	0	0		
	Physical Damage ignores 8% of the target's armor						
	Reduces damage taken from monsters by 2						
	Deals 6 true damage to any minion or monster that attacks you						
	Reduces damage taken by 3%						
	Active: Deals 460+ (level·30) true damage to target enemy minion (cooldown =70). Get 10 gold						
Other	Each autoattack deal 2.5+ (0.5·level) additional magic damage and heal Warwick for the same amount. Each successive attack against the same target will stack up to a maximum of 3 stacks.						
Miscellaneous	Active: For 10 seconds, Warwick increases his attack speed by 40 and increases						
Statistics and	the attack speeds of all allies within range by 20 (cooldown 24 mana cost=35)						
Actives	Active: Warwick dealsActive: Warwick dealsActive:75+100%AP magic125+100%AP magic175+1damage to targetdamage to targetdamage to targetdamage to targetdamage to targetdamenemy and healsenemy and healsenerhimself for 80% of thehimself for 80% of thehimselfdamage dealtdamage dealtdamage dealt(cooldown 10;(cooldown 9;(cooldown 9;mana=70)mana=80)r		Active: Warwick deals 175+100%AP magic damage to target enemy and heals himself for 80% of the damage dealt (cooldown 8; mana=90)				
	Damage dealt to monsters increased by 10%						

E2: Starting Items and Madred's Razor

	Level 2	Level 3	Level 4	Level 5			
Health	680	782	884	986			
Health Regeneration	8.65	9.45	10.25	11.05			
Mana	250	280	310	340			
Mana Regeneration	8.30	8.90	9.50	10.10			
Attack Damage	64.9	68.9	72.9	77.0			
Attack Speed	0.808	0.827	0.847	0.866			
Armor	62.7	66.2	69.7	73.2			
Magic Resist	32.5	33.8	35.0	36.3			
Movement Speed	345	345	345	345			
Range	125	125	125	125			
Armor Penetration	8.1	8.1	8.1	8.1			
Magic Penetration	6.1	6.1	6.1	6.1			
Ability Power	3.1	4.6	6.1	7.7			
Lifesteal	0	0	0	0			
	Physical Damage ignores 8% of the target's armor						
	Reduces damage taken from monsters by 2						
	Deals 6 true damage to any minion or monster that attacks you						
	Reduces damage taken by 3%						
	Active: Deals 460+ (level 30) true damage to target enemy minion						
	(cooldown =70). Get 10 gold						
	Each autoattack deal 2.5+ (0.5·level) additional magic damage and heal						
	Warwick for the same amount. Each successive attack against the same target						
Other Miscellaneous	will stack up to a maximum of 3 stacks.						
Statistics and Actives	Active: For 10 seconds, Warwick increases his attack speed by 40 and increases						
	the attack speeds of all allies within range by 20 (cooldown 24 mana cost=35)						
	Active: Warwick deals	Active: Wa	rwick deals	Active: Warwick deals			
	75+100%AP magic	125+100%	6AP magic	175+100%AP magic			
	damage to target enemy	damage to t	arget enemy	damage to target enemy			
	and heals himself for	and heals	himself for	and heals himself for			
	80% of the damage	80% of th	e damage	80% of the damage			
	dealt (cooldown 10;	dealt (co	oldown 9;	dealt (cooldown 8;			
	mana=70)	mana	a=80)	mana=90)			
	basic attacks deal 60 bonus magic damage to monsters on hit						

	Level 3	Level 4	Level 5			
Health	782	884	986			
Health Regeneration	9.45	10.25	11.05			
Mana	280	310	340			
Mana Regeneration	8.90	9.50	10.10			
Attack Damage	78.9	82.9	87.0			
Attack Speed	0.827	0.847	0.866			
Armor	66.2	69.7	73.2			
Magic Resist	33.8	35.0	36.3			
Movement Speed	345	345	345			
Range	125	125	125			
Armor Penetration	8.1	8.1	8.1			
Magic Penetration	6.1	6.1	6.1			
Ability Power	4.6	6.1	7.7			
Lifesteal	0.1	0.1	0.1			
	Physical Damage ignores 8% of the target's armor					
	Reduces damage taken from monsters by 2					
	Deals 6 true damage to any minion or monster that attacks you					
	Reduces damage taken by 3%					
	Active: Deals 460+ (level·30) true damage to target enemy minion (cooldown =70). Get 10 gold					
	Each autoattack deal 2.5+ (0.5·level) additional magic damage and heal					
Other Miscellaneous Statistics and Actives	Warwick for the same amount. Each successive attack against the same target will stack up to a maximum of 3 stacks.					
	Active: For 10 seconds, Warwick increases his attack speed by 40 and increases the attack speeds of all allies within range by 20 (cooldown 24 mana cost=35)					
	Active: Warwick deals 125+100%AP magic damage to target enemy and heals himself for 80% of the damage dealt (cooldown 9; mana=80)Active: Warwick deals 175+100%AP magic damage to target enemy and heals himself for 80% of the damage dealt (cooldown 8; mana=90)					
	Dasic attacks deal bo bonus magic damage to monsters on hit					

E3: Starting Items, Madred's Razor and Vampiric Scepter