Optimal Placement of Hand Sanitizers at Carnegie Mellon University

A Proposal for University Administrators and Officials

Proposal prepared by Antonios Tavlarakis and Akshay Upadhyay.

Executive Summary:

Over the past year, the H1N1 virus has become a major threat to global human health. Governments around the world are taking necessary actions and preventative measures to avert a health crisis reminiscent of the influenza pandemic in the early 1920's. Here at home, the Carnegie Mellon Health Services department has attempted to mitigate the spread of the disease with a three pronged approach; quarantining individuals with the virus, providing prevention services (i.e. vaccination clinics) and placing hand sanitizers throughout campus. The purpose of these hand sanitizers is to reduce hand-hand or hand-surface-hand transmission of the virus. In this paper, multiple mathematical approaches were taken to determine a more optimal placement of hand sanitizers throughout some buildings on campus. Ultimately, we found a more optimal placement of hand sanitizers which we hope will aid university officials in increasing accessibility to sanitization and thereby improve health services in both staving-off infections and decreasing expenses by investing in prevention. It was determined that areas with higher traffic, as opposed to entrances/exits, are better locations to place hand sanitizers, and a more thorough/precise placement will be discussed.

Introduction:

In June of 2009, the World Health Organization (WHO) officially declared the H1N1 virus a global pandemic[i]. In fact, in December 2009, the Center for Disease Control released statistics that approximately 10,000 Americans had died as a direct result of the H1N1 virus[ii]. The pandemic flu is not the only troubling issue. The United States and countries around the world are currently facing economic problems ranging from lowered consumer spending to increases in unemployment figures. In such turbulent economic times, a pandemic of any proportion could continue to cripple an already unstable global economy. As a result of these potentially severe ramifications, governments around the world have taken multiple initiatives to protect their citizens.

In the United States, the Center for Disease Control (CDC) is the branch of the government which handles all concerns related to any disease. Lately, it has been spearheading multiple initiatives to combat H1N1. One such achievement includes developing a test kit to determine whether someone is infected with the virus.[iii] While the CDC is a powerful government branch, it is an organization that operates at the national level and thus heavily relies on the efforts of small communities and local governments in order to launch a successful effort. At Carnegie Mellon University (CMU), the Health Services department is responsible for any issues pertaining to H1N1. While Health Services handles the quarantining of infected students and running vaccine clinics, it has also taken the initiative of installing hand sanitizers. At CMU, the logic is that preventing the H1N1 infection will lower the number of people quarantined, creating a healthier atmosphere. Such endeavors however, cost money, and ideally we would like to spend the least amount of money while providing the same quality of service. This is especially true in economically troubling times, when organizations constantly try every cost-cutting technique. Our report focuses on trying to place the hand sanitizers in more optimal positions to target more individuals, while at the same time trying to minimize CMU's expenses. Multiple mathematically sound procedures were developed and used to solve the task at hand. For this paper, we perform these computations on two of the most frequented academic buildings, Baker Hall and Wean Hall.

Mathematical Modeling:

Integer Programming Formulation

In order to begin explaining the mathematical overview of the problem, it is necessary to introduce a few basic concepts from graph theory.

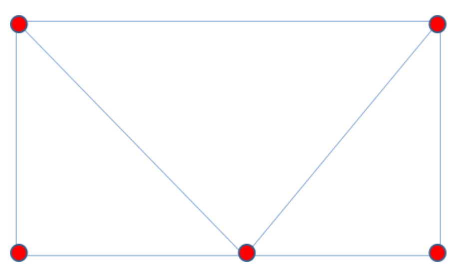


Figure 1: Simple Graph

The simple graph presented above is shown to portray a simplified graph that will be used in the floor plans for Carnegie Mellon buildings. The red nodes are used to denote potential locations where hand sanitizers can be placed. The edges shown above are used to denote hallways that students and faculty take to get from one place to another. Two or more adjoined edges which do not create a cycle are paths. In our model, some edges are merged together to form paths.

Finally, it is important to note that every edge in the graphs has an associated edge weight. In our model, edge weights are influenced by the amount of traffic observed in each hallway. To account for this traffic, we created a categorical variable to denote edge weights, whose values are summarized in the table below.

Traffic Level	Weight
Little or No Traffic	1
Light Traffic	2
Moderate Traffic	3
Heavy Traffic	4
Rush Hour	5

As can be seen, edges where there is little to no traffic, have a rating of one. Consequently, the most frequented hallways and corridors are given the highest rating of 5. It is important to state here, that every node has the weight of the corresponding edge it is located on. More importantly, a node located at the junction of two or more edges of different weights, has a weight of the average of all the edges it connects to. For instance, suppose a node is on one edge of weight 5 and another edge of weight 4. In this case, the node has a weight of 4.5.

The variables in our integer program represent every location, or node, where it is possible to insert a hand sanitizer. Consequently, the variable is binary and takes on the values zero or one depending on whether at a hand sanitizer is placed at a specific location. Our primary objective for this problem is to minimize total expenditures for CMU while our secondary objective is to maximize the number of people that have access to hand sanitizers. Since we are minimizing, having coefficients with the highest values is unwise because we want the nodes with highest traffic ratings to have higher priority. Hence, we take the multiplicative inverse of the edge weight, in which case the nodes with highest weight have highest priority of being selected in order to minimize the objective function. The formulation of the minimizing function is presented below:

$$\text{Minimize } z = \sum_{i=1}^{n} \frac{x_i}{\omega_i}$$

We now move onto a discussion of the constraints. In this model, we have multiple constraints that we require to be met. The first one is that the sum of the constraints must be larger than or equal to three times the average weight on each floor.

$$\sum_{i=1}^{n} x_i \geq 3 \cdot \left\lceil \frac{\sum_{i=1}^{n} \omega_i}{n} \right\rceil$$

The idea behind this constraint is to provide a lower bound on the number of nodes selected. In other words, we want minimum number of hand sanitizers on each floor of the respective building that we are looking at. There are an additional *m* more constraints that are used to provide an upper barrier on the number of edges on each node. In our model, we do not want more than two sanitizers in the same corridor.

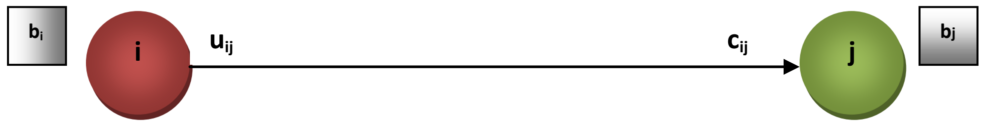
$$\sum_{i=1}^{k_j} x_i \leq 2$$

Not having these constraints would results in selected nodes with the highest traffic which could yield the placement of many hand sanitizers on every node on an edge with the highest traffic. Due to a budget constraint that CMU faces, this is not a feasible solution. We also have constraints for every entrance. Indeed. High volumes of traffic at entrances made us believe that it was imperative to place a hand sanitizer at every entrance to the building. Consequently, the variables denoting these locations would simply take the value 1. The final constraint, and one that we have previously mentioned, is that every variable must take on the values of zero or one.

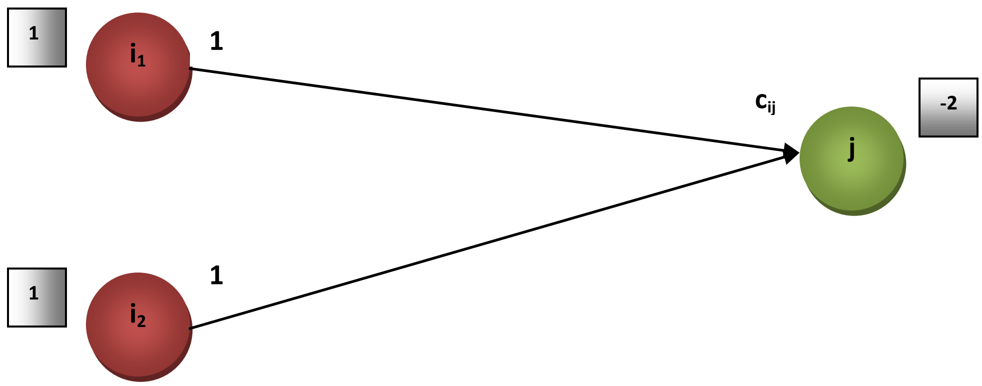
Network Model Formulations

The objective of our network model is to maximize the amount of people that have access to hand sanitizers. This objective is constrained on the cost of installing a hand sanitizer at a specific location. Also, we did not have any restrictions on which nodes had to be selected as we did in the integer programming model. That is, entrances were not needed to have hand sanitizers.

Let’s begin with some important terminology. Let G = (V, E) be a directed network defined by a set V of vertexes and set E of edges. For each edge (i, j) ∈ E we associate a capacity u_{ij} that denotes the maximum amount that can flow on the edge. Each edge (i, j) ∈ E also has an associated cost c_{ij} that denotes the cost per unit flow on that edge. We associate with each vertex i ∈ V a number b_i. The value represents supply/demand of the vertex. If $b_i > 0$, node i is a supply node; if $b_i < 0$ node I is a demand node. For simplicity, we’ll call G a transportation network and write G = (V, E, u, c, b) and show all the network parameters explicitly.



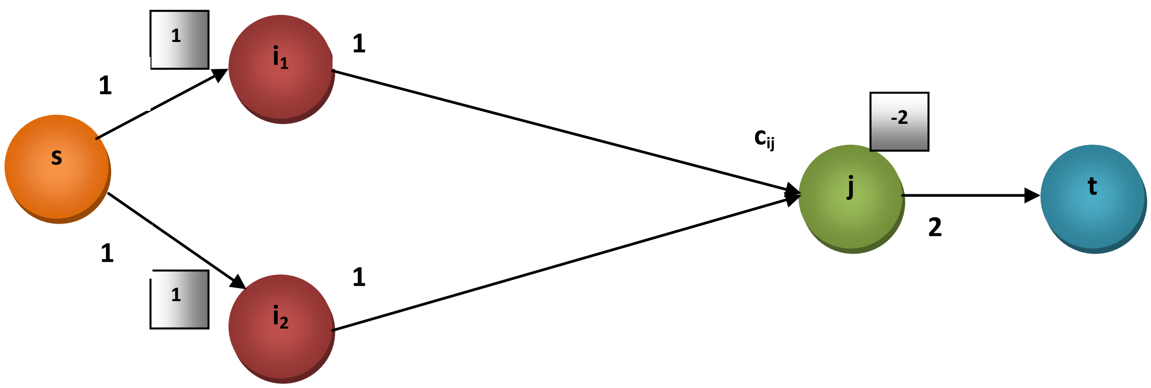
The figure above is an example of an edge (i, j) ∈ E. We have a supply vertex i ∈ V and a demand vertex j ∈ V. The edge has two numbers, capacity (on the left) and cost (on the right).



The figure above is an example of our transportation network. The nodes i ∈ V (on the left) represent potential locations for hand sanitizers. The nodes j ∈ V represent the paths in our model. In this network we have two supply vertices (on the left). Each has a supply value of 1. This is to implement that we can only have one hand sanitizer at each location. We also have one demand vertex in this network (on the right). It has a demand value of -2 in order to implement that we cannot have more than two hand sanitizers on each path. Each edge has two numbers, capacity and cost. In our model, all of the capacities are equal to 1 and are assigned to each location. This is because each location corresponds to a specific path only once. The costs, in our model, correspond to the traffic values that we assigned earlier to each edge, shown in the following table. The reason why we used this table to derive the costs of each path was to ensure that we minimize costs in our model. The path with the highest traffic will have the lowest cost to install a hand sanitizer, while the path with the lowest traffic will have the highest cost to install a hand sanitizer.

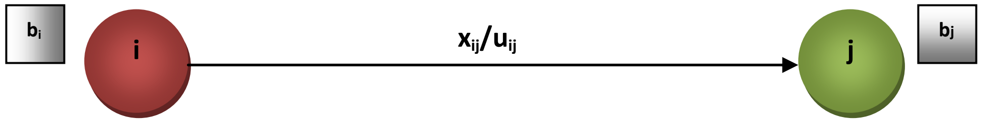
Traffic	Cost
1	5
2	4
3	3
4	2
5	1

Let us introduce source nodes s and a sink node t. For each node i ∈ V with $b_i > 0$, we add a source arc (s,i) to G with capacity b_i and cost 0. For each node j ∈ V with $b_i < 0$, we add a sink arc (i,t) to G with capacity $-b_j$ and cost 0.



The new network is the transformed network seen above. Now, we solve a maximum flow problem from s to t. While the primary objective of the integer programming model was to minimize the cost, the primary objective of our network model is to maximize the flow, that is, maximize the number of people that have access to a hand sanitizer. The secondary objective of this model is to minimize cost.

To begin solving this problem we must select the path with the absolute minimum cost. The rule of thumb that we will use for breaking ties in the paths is that we will select the node with the lowest lexicographical index. The next step is to select up to two location nodes corresponding to the selected path nodes. The rule of thumb for breaking ties in the locations is that we will select the node with the lowest numerical index. We continue doing this for up to 2m iterations where m is the number of path nodes. We stop the method when one or both of two conditions are satisfied. The first condition is that all of the demand has been satisfied. The second condition is that for the path nodes for which the demand has not been met, there are no more corresponding location nodes that haven't already been used.



The example of the edge above illustrates the notation of the results of our network model. We let x_{ij} denote the flow through each edge and let u_{ij} denote the capacity of each edge. This is the notation for the flow of each edge in the model including edges connecting the source node with the location nodes and the edges connecting the path nodes with the sink node.

Model Assumptions for Network Formulation

Assumption 1: *All data (u_{ij}, c_{ij}, b_i) are integral.*

Unlike the integer programming model, in which we had rational coefficients of the objective function, we assume that all the numbers in our network model are integers. We use the same ranking criteria as before. But, we convert the rational rankings of each node in the integer program to integral rankings in the form of costs in the network model.

Assumption 2: *The network is directed.*

We need to assume that our network model consists of a directed network. This is because of the context of the problem that we are solving. This is not a typical maximal flow problem in which we determine the maximum flow from a source to a sink. In our network model, we have two sets of nodes that have a specific relation between them.

Assumption 3: *All costs associated with edges are nonnegative.*

We assume that all costs associated with the edges are nonnegative. This is because the installation of hand sanitizers has a cost to it. The benefit of it has already been taken into account through the ranking that we have used.

Discussion/Results:

Integer Programming Model Results:

Now that the modeling procedures have been discussed, we can present the results. Keep in mind that there are two models that are incorporated into this paper. We present these results independently. Displayed first, in Figures 1, 2, 3, 4, 5, and 6, are the results from the Integer

Programming model. As was mentioned in the mathematical modeling section, each location where an entrance/exit is present (denoted by blue squares) is guaranteed to have a sanitizer present as a results of our constraints. As was expected, the formulation selected points based on those with highest weights and subject to the constraints. Interestingly, some of these solutions are not unique. For instance, in figure #, the circle around the pushpin serves to show us that the solution is not unique. The reason for this is that the location could be anywhere along its corresponding edge (excluding the endpoints). For instance, there would be no impact to the value returned by the objective function, nor any violation of the constraints should the optimal placement be shifted one location to the right. We do note that the absence of uniqueness is not necessarily a disadvantage of our formulation. In fact, with respect to our application, it is particularly favorable as it allows flexibility in placement. If one location may be hard, unsuitable, or non-cost effective to place a hand sanitizer we have the ability to look for alternative locations to place a hand sanitizer without sacrificing the value returned by the objective function.

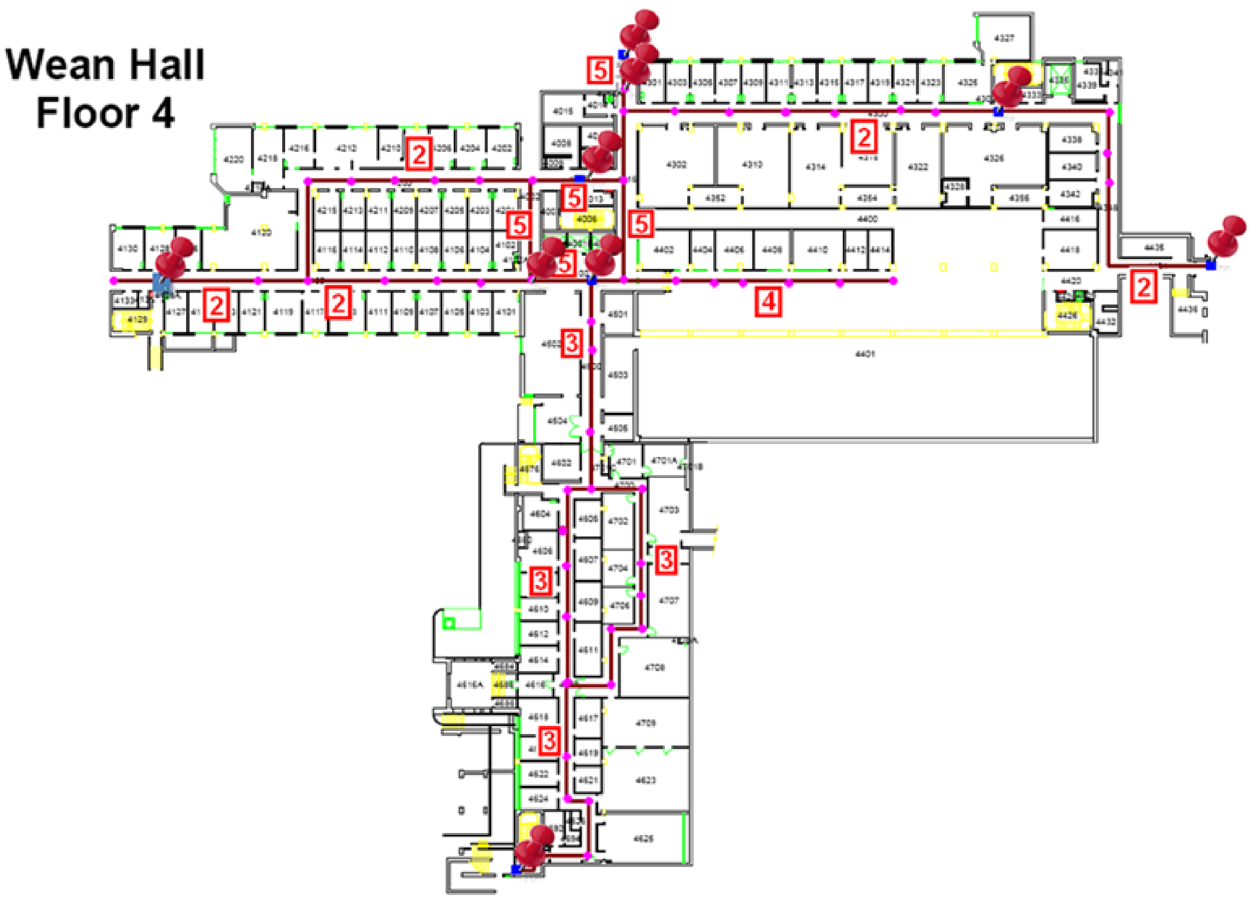


Figure 2: Wean Hall Fourth Floor- IP Optimal Hand Sanitizer Placement

Optimal Placement of Hand Sanitizers at Carnegie Mellon University

Figure 3: Wean Hall Fifth Floor- IP Optimal Placement of Hand Sanitizers

Figure 4: Wean Hall Eighth Floor- IP Optimal Placement of Hand Sanitizers. Note that due to the similar structure and traffic of floors 6 and 7 to 8, we expect placement to be very similar. We omit presenting results for Floors 6 and 7.

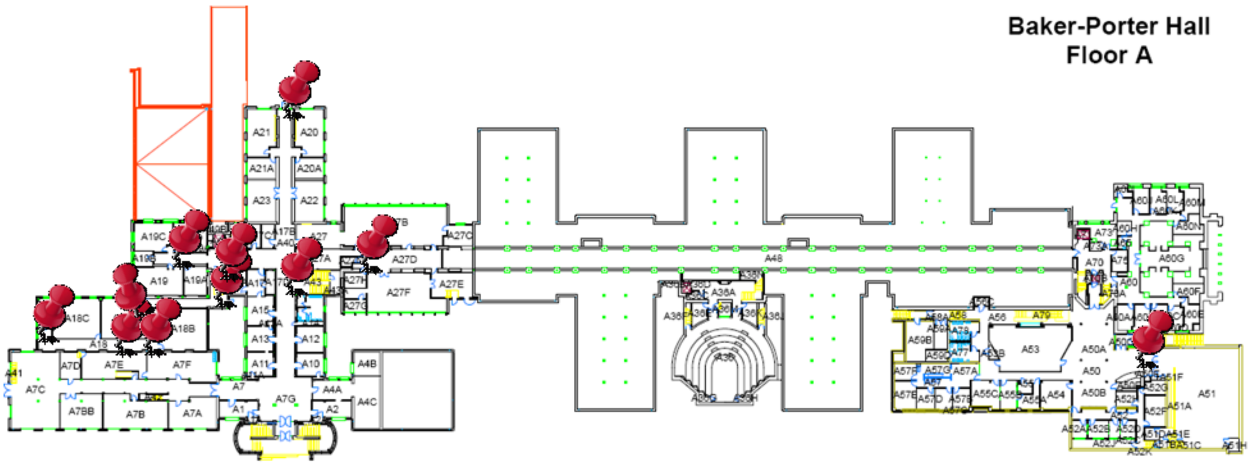


Figure 5: Baker Hall Floor A- IP Optimal Placement of Hand Sanitizers

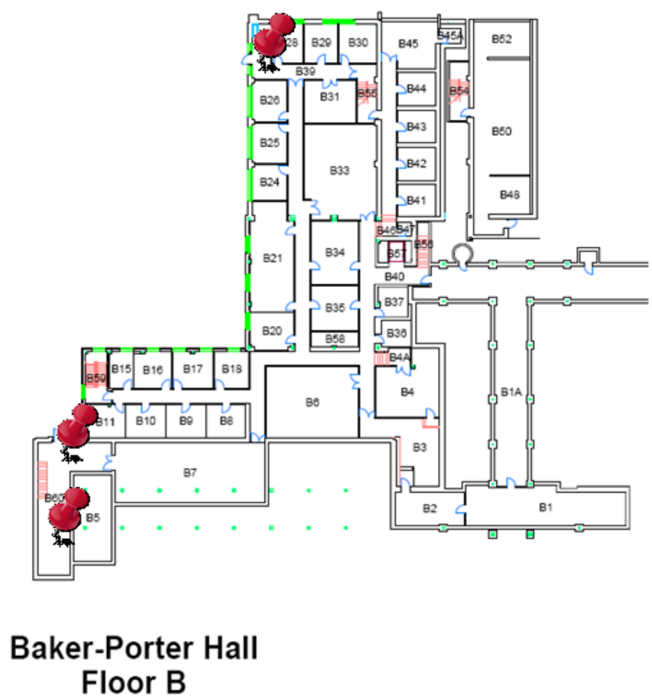


Figure 6: Baker Hall Floor B- IP Optimal Placement of Hand Sanitizers

Figure 7: Baker Hall Floor 1- IP Optimal Placement of Hand Sanitizers

Networking Model Results:

We used our network model to find a solution to the two most frequented floors in the buildings. These floors were the first floor of Baker/Porter Hall and the fifth floor of Wean Hall. We specifically selected these two floors because they both had multiple entrances. We wanted to see how not having these constraints affects the solution. In addition, the average traffic all the nodes on each of these two floors where the two highest among all of the floors that we investigated.

The solutions for Baker Hall Floor 1 using the network model were nodes 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 31, 32, 33, 34, 35, 36, 47, 48, 53, 54, 57, 58, 60, 62, 63, 64, 65, and 68. The total count is 36 hand sanitizers; significantly more than the solution obtained via the integer programming model (12).

Despite this fact, we see that there were some additional nodes found as solutions in the integer programming model that were not present in the network model solution. But, only one of these additional locations selected in the integer programming was an entrance. Therefore, we can conclude that having the constraint that each entrance must have a hand sanitizer does not drastically affect the results. This may be due to the fact that the entrances are usually high-traffic locations and so favorable for selection by the models. The difference in the results may be due to the lack of uniqueness of solutions for each model algorithm employed.

The solutions for Wean Hall Floor 5 using the network model were nodes 1, 2, 8, 9, 10, 11, 12, 18, 19, 20, 21, 28, 29, 30, 31, 33, and 34 (19 hand sanitizers). Our solution of the integer programming model yielded 13 hand sanitizers. There were few differences in the two solutions. Again, there is one solution, an entrance, included in the integer programming solution but excluded from the network model solution. But, this must be, as we already mentioned, due to the fact that there are multiple potential solutions.

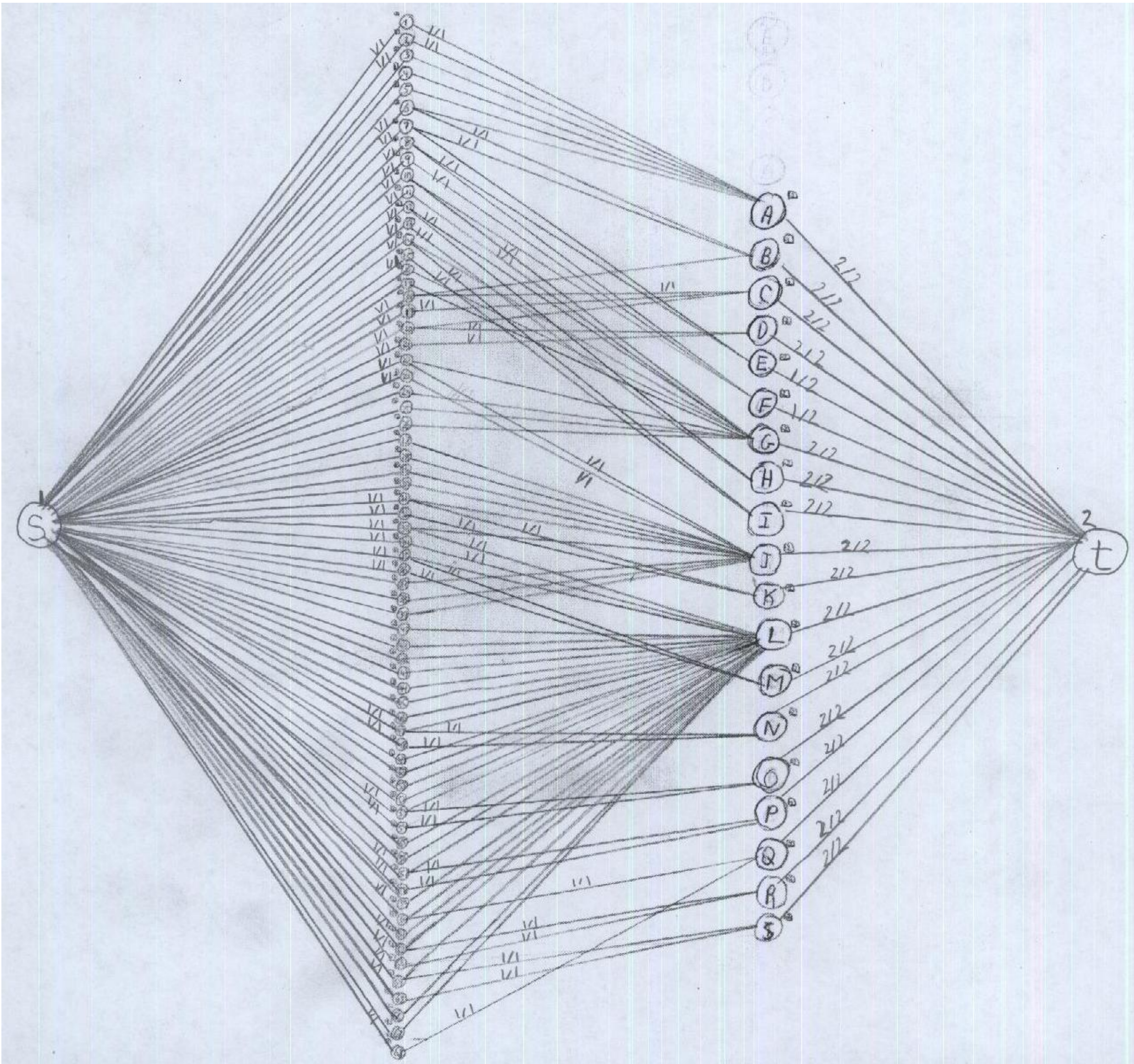
Conclusion

In this paper, we investigated the optimal placement of hand sanitizers in an attempt to increase their availability. Two mathematical techniques were employed; Integer Programming and Network Models. A comparison of the two results revealed that the results were similar but not identical. We note that due to the absence of uniqueness, the differences in the results may vary due to tie-breaking procedures. Additionally, there are more hand sanitizer locations picked using the network model as opposed to the integer programming model. This, however, was due to the model definitions. In conclusions, we believe that both models yield similar results.

There is lots of potential work that can be done in the context of the problem presented in this paper. Our model did have some shortcomings, the major one being that we had to create multiple points for the graphs. The shortcoming here is that between two locations there may be a more optimal selection. To take this into consideration, we should consider using more points or apply the Weber Method which would allow us to do continuous optimal placement (our model uses discrete optimal placement). In some of our results, specifically Figure 5, there are many nodes that are located next to each other. We can implement more constraints that prevent the integer program from selecting two that are side by side. Such a constraint would impose a penalty for putting two hand sanitizers next to each other. For the networking problem, we could have allowed more than one hand sanitizer at each location, which would have changed the capacity of each edge depending on how many we wanted to allow.

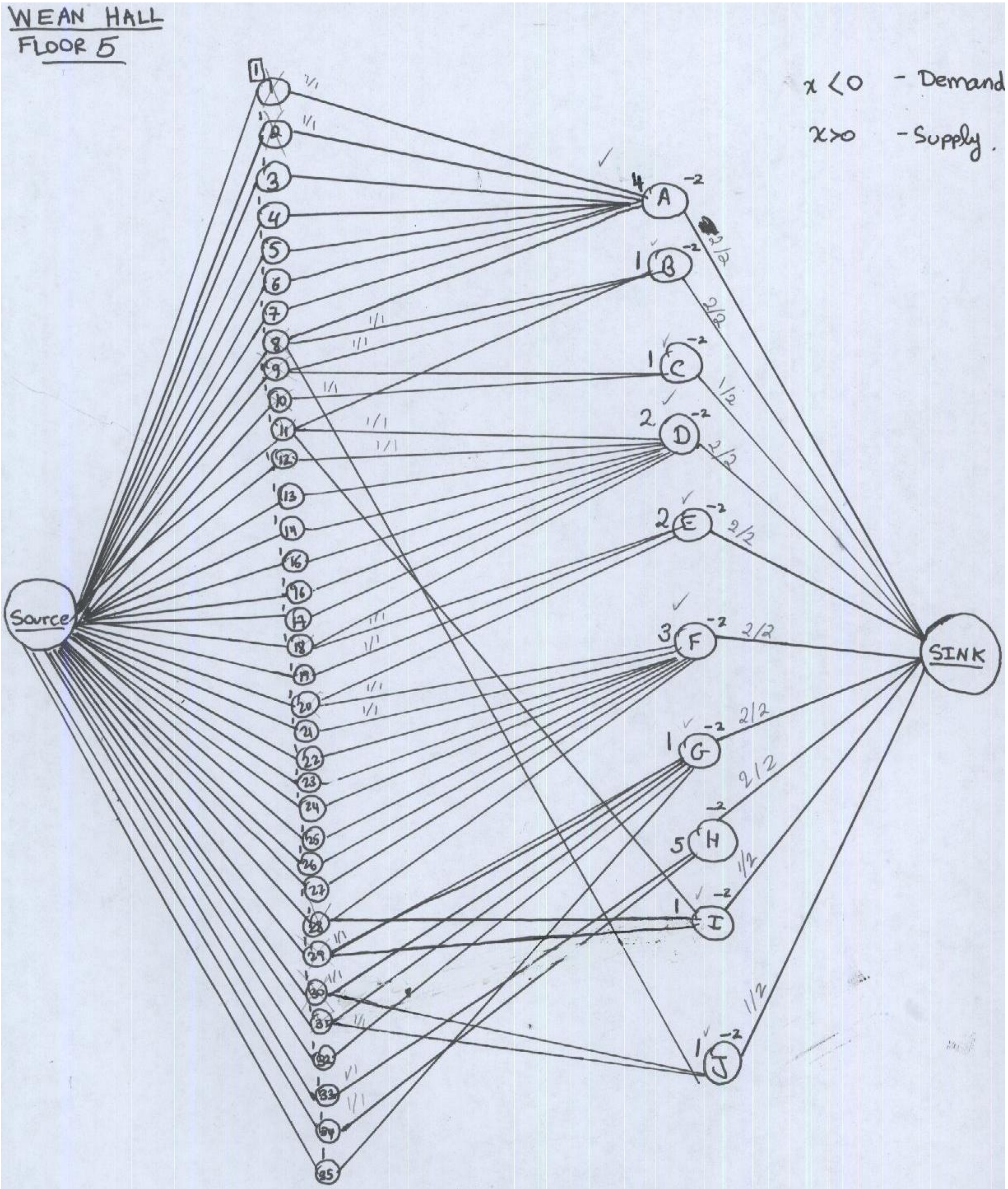
Optimal Placement of Hand Sanitizers at Carnegie Mellon University

Appendix



Appendix 1: Baker Hall Floor 1 Network Solution

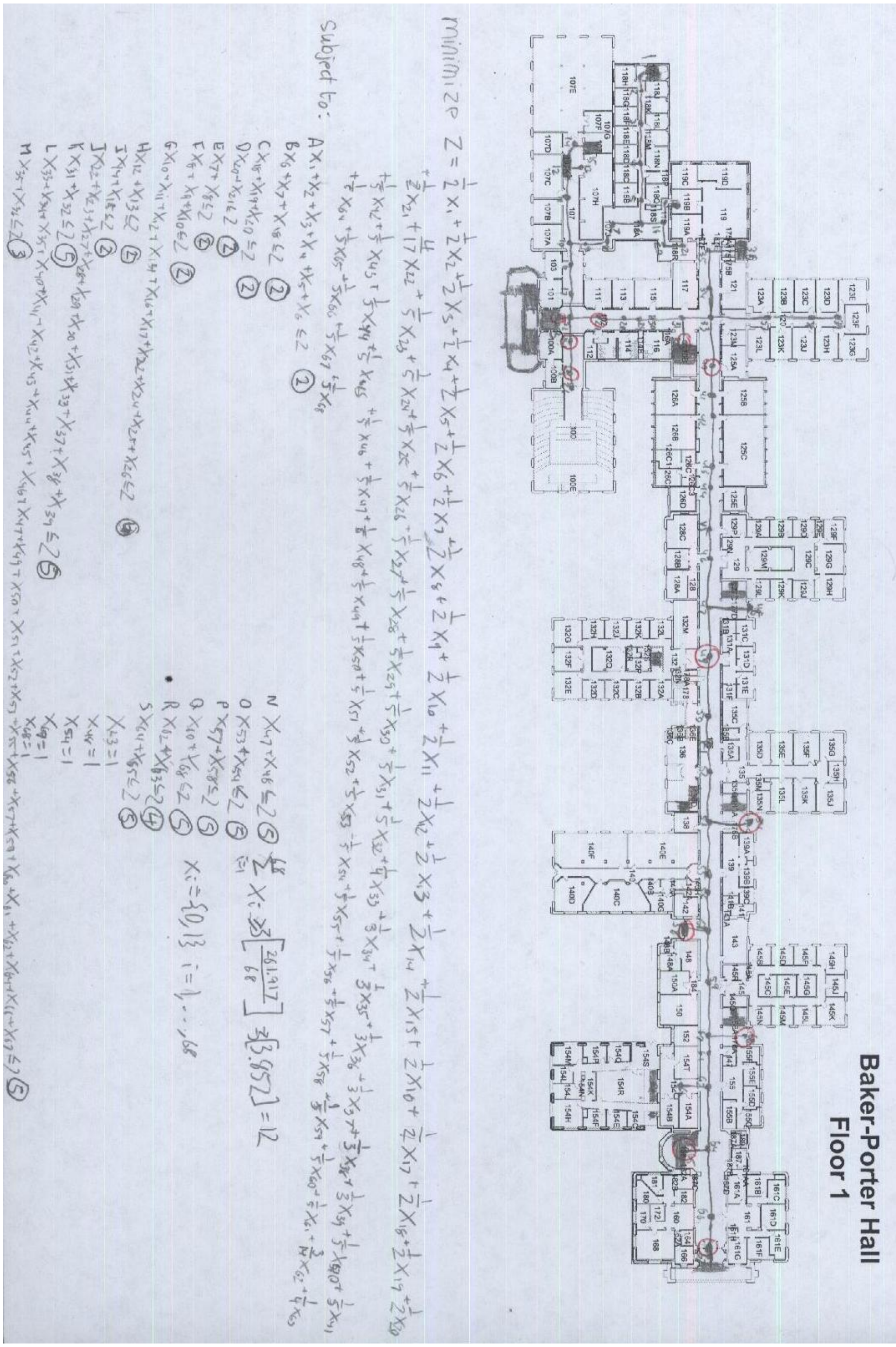
Optimal Placement of Hand Sanitizers at Carnegie Mellon University



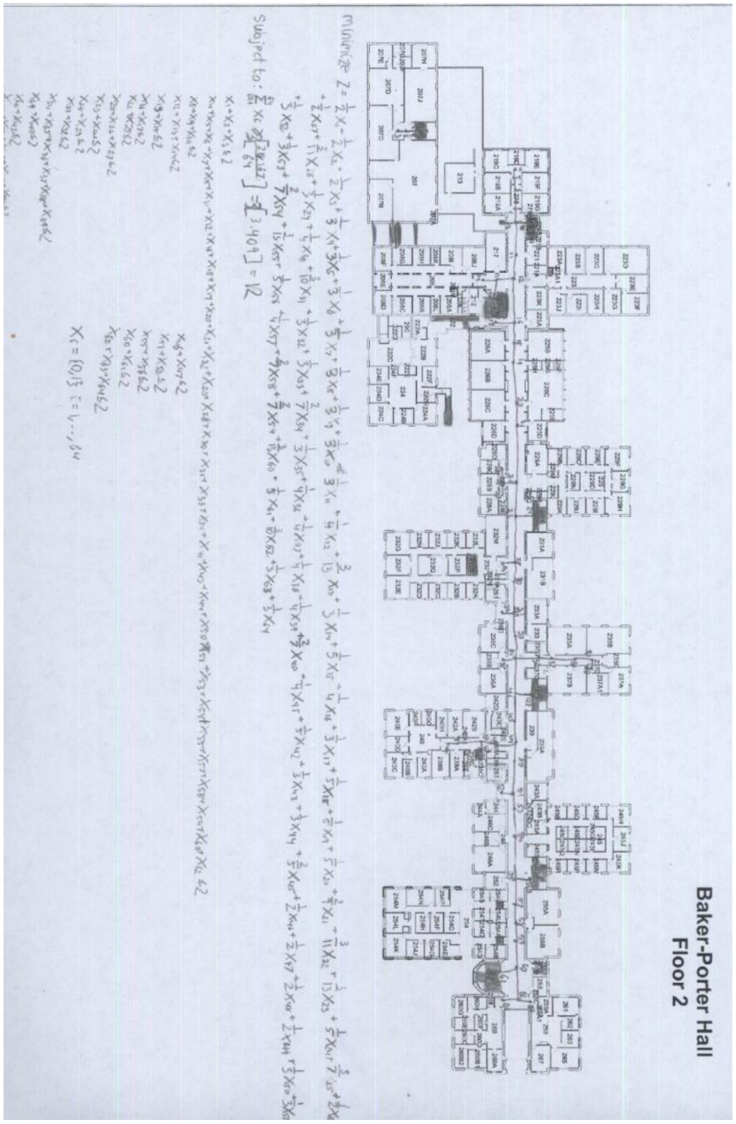
Appendix 2: Wean Hall Floor 5 Network Solution

Optimal Placement of Hand Sanitizers at Carnegie Mellon University

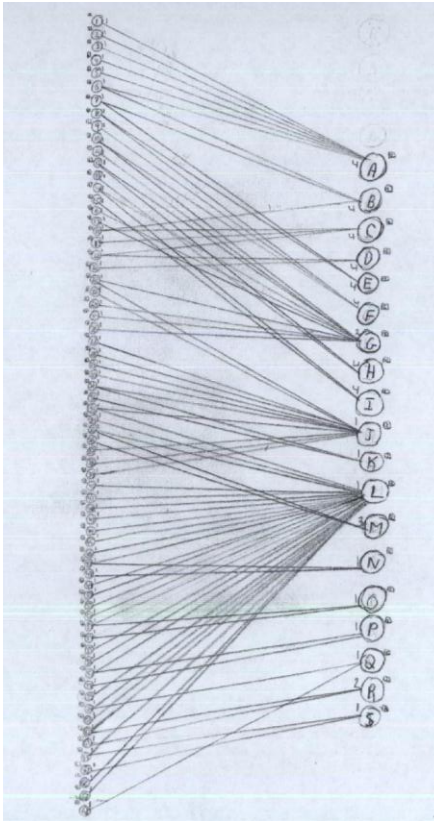
Integer Programming Formulations



Appendix 3: IP Formulation for Baker Hall Floor 1



Appendix 4: IP Formulation for Baker Hall Floor 2



Appendix 5: Partial Formulation for Network Problem

---

[i] http://www.who.int/mediacentre/news/statements/2009/h1n1_pandemic_phase6_20090611/en/index.html
[ii] http://www.nytimes.com/2009/12/11/health/11flu.html
[iii] http://www.cdc.gov/h1n1flu/updates/us/