Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 3: Due Wednesday October 9.

Q1 Analyse the following inventory system and derive a strategy for minimising the total cost. There are n products. Product i has demand λ_i per period and no stock-outs are allowed. The cost of making an order for Q units of a mixture of products is AQ^α where $0 < \alpha < 1$. The inventory cost is I times $\max\{L_1, L_2, \ldots, L_n\}$ per period where L_i is the average inventory level of product i in that period.

Q2 Give an algorithm to solve the following scheduling problem. There are n jobs labelled $1, 2, \ldots, n$ that have to be processed one at a time on a single machine. There is an acyclic digraph $D = (V, A)$ such that if $(i, j) \in A$ then job j cannot be started until job i has been completed. The problem is to minimise $\max_j f_j(C_j)$ where for all j, f_j is a monotone increasing. As usual, C_j is the completion time of job j. This is distinct from its processing time p_j.

Q3 Convert the following into a standard assignment problem. We have a bipartite graph with bipartition $A = \{a_1, a_2, \ldots, a_m\}, B = \{b_1, b_2, \ldots, b_n\}$. An assignment now is a set of edges M such (i) a_i is incident to exactly r_i edges of M for $i = 1, 2, \ldots, m$ and (ii) b_j is incident to exactly s_j edges of M for $j = 1, 2, \ldots, n$. Here $\sum_i r_i = \sum_j s_j$. The cost of edge (a_i, b_j) is $c(i, j)$ and the cost of an assignment M is $\sum_{e\in M} c(e)$. The objective is to find a minimum cost assignment.