Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 2: Due Monday September 23.

Q1 Formulate the following as an integer program: There are n students and exams $E_1, E_2, \ldots, E_m \subseteq [n]$ need to be scheduled. Student i takes exams $S_i \subseteq [m]$. There are s rooms available and each room can hold r students. The rules are
(i) A student must not be asked to take more than one exam per day;
(ii) Several different exams can be held in the same room provided there is capacity in the room to hold the students.
(iii) No student has to take 3 exams in 3 consecutive days.

The problem is to minimise the number of days needed to carry out all of the exams.
(Hint: let $x_{i,j,k} = 1$ iff exam i takes place in room j on day k and $y_i = 1$ if there is an exam on day i.)

Q2 The government has asked for and received bids on m construction projects from each of n firms. No firm will be awarded more than one contract and for political reasons no more than p large contracts are to go to foreign firms. Projects $1, 2, \ldots, \ell$ are large and firms $1, 2, \ldots, f$ are foreign. If $b_{i,j}$ is the bid by firm i for project j, which bids should be accepted to minimise the total cost?

Q3 Solve the following problem by a cutting plane algorithm:

$$\begin{array}{lrcrcrl} \text{minimise} & 4x_1 & + & 5x_2 & +3x_3 & & \\ \text{subject to} & & & & & & \\ & 2x_1 & + & x_2 & -x_3 & \geq & 2 \\ & x_1 & + & 4x_2 & +x_3 & \geq & 13 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$