Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 1: Due Monday September 9.

Describe a Dynamic programming solution to the following problems:

Q1 Solve the following knapsack problem:

$$\begin{array}{lrcrcrcl} \text{maximise} & 2x_1 & + & 4x_2 & + & 10x_3 & & \\ \text{subject to} & & & & & & & \\ & 2x_1 & + & 3x_2 & + & 6x_3 & \leq & 15 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Q2 Consider a 2-D map with a horizontal river passing through its center. There are n cities on the southern bank with x-coordinates $a(1)...a(n)$ and n cities on the northern bank with x-coordinates $b(1)...b(n)$. You want to connect as many north-south pairs of cities as possible with bridges such that no two bridges cross. When connecting cities, you can only connect city i on the northern bank to city i on the southern bank. Construct a Dynamic Programming solution to this problem. (You can assume that $a(1) < a(2) < \cdots < a(n)$, but you **cannot** assume that $b(1) < b(2) < \cdots < b(n)$. If both sequences are increasng, then the problem is trivial).

Q3 A company manufactures two products A and B at a certain facility. The demands for the products are $a_i, b_i, i = 1, 2, \ldots, n$ over the next n periods. The cost of making x of either product is $c(x)$ and there is room to store H in total of the two products. Cleaning problems require that only one product can be manufactured in any one period and that the same product cannot be manufactured for more than 3 consecutive periods. Assume that at the beginning of period one there is $H/2$ of each product in storage. The problem is to minimise total cost, given that all demands must be met.