Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 2: Due Friday September 27.

Q1 Can the following shortest path problem be solved by the Dijkstra algorithm? The edges of a digraph are colored Red, Blue and Green. Suppose edge lengths are non-negative, but a path can have at most r Red edges, b Blue edges and no Blue edge can be followed by a Green edge. Give an explicit definition of path length.

Q2 Convert the following into a standard assignment problem. We have a bipartite graph with bipartition $A = \{a_1, a_2, \ldots, a_m\}, B = \{b_1, b_2, \ldots, b_n\}$. An assignment now is a set of edges M such (i) a_i is incident to exactly r_i edges of M for $i = 1, 2, \ldots, m$ and (ii) b_j is incident to exactly s_j edges of M for $j = 1, 2, \ldots, n$. Here $\sum_i r_i = \sum_j s_j$. The cost of edge (a_i, b_j) is $c(i, j)$ and the cost of an assignment M is $\sum_{e \in M} c(e)$. The objective is to find a minimum cost assignment.

Q3 Let $G = (A, B, E)$ be a bipartite graph. Let $I \subseteq B$ be independent if G contains a matching M that is incident with every vertex in I. Show that the independent sets form a matroid.
Hint: consider the action of augmenting paths.