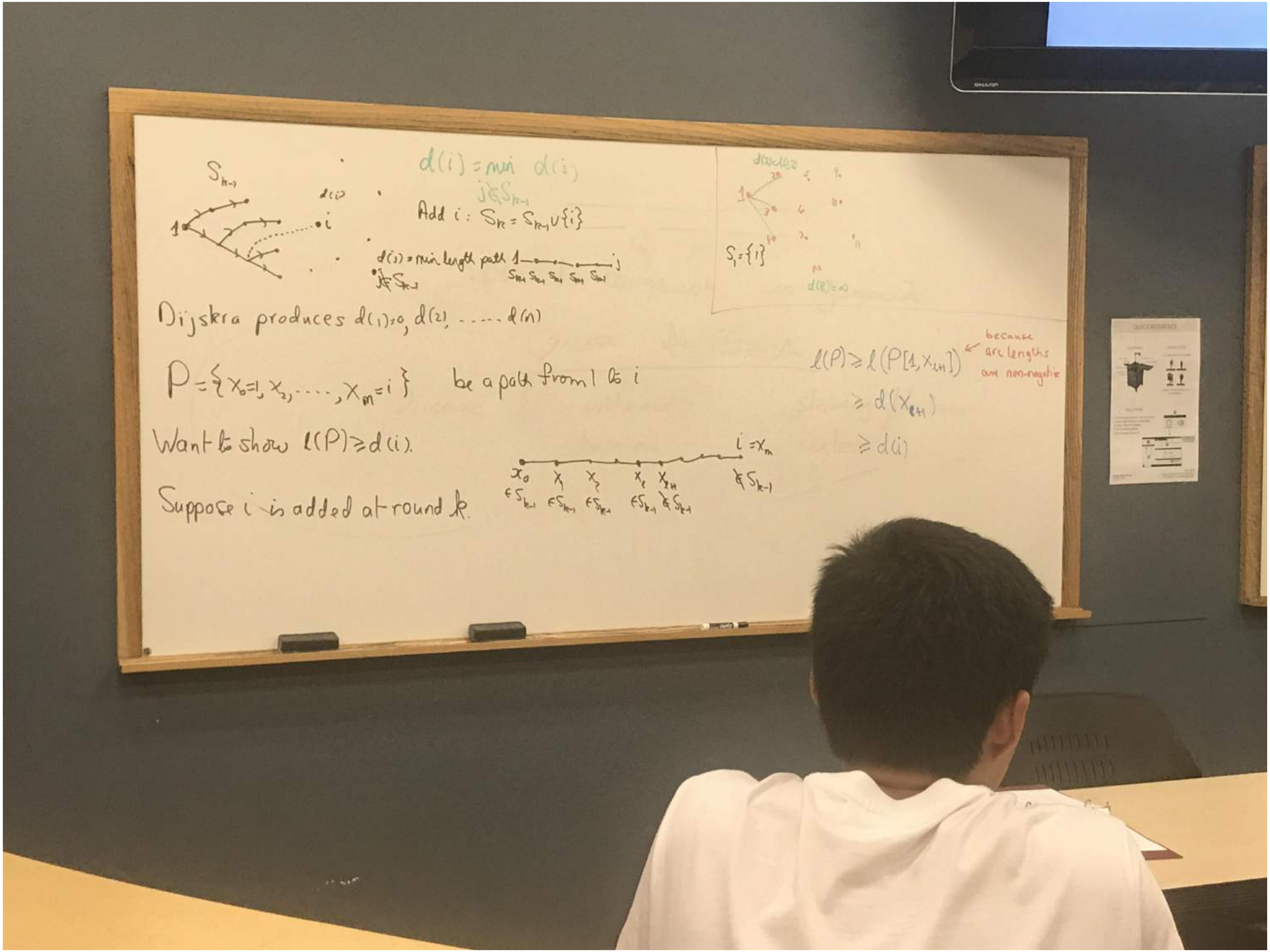
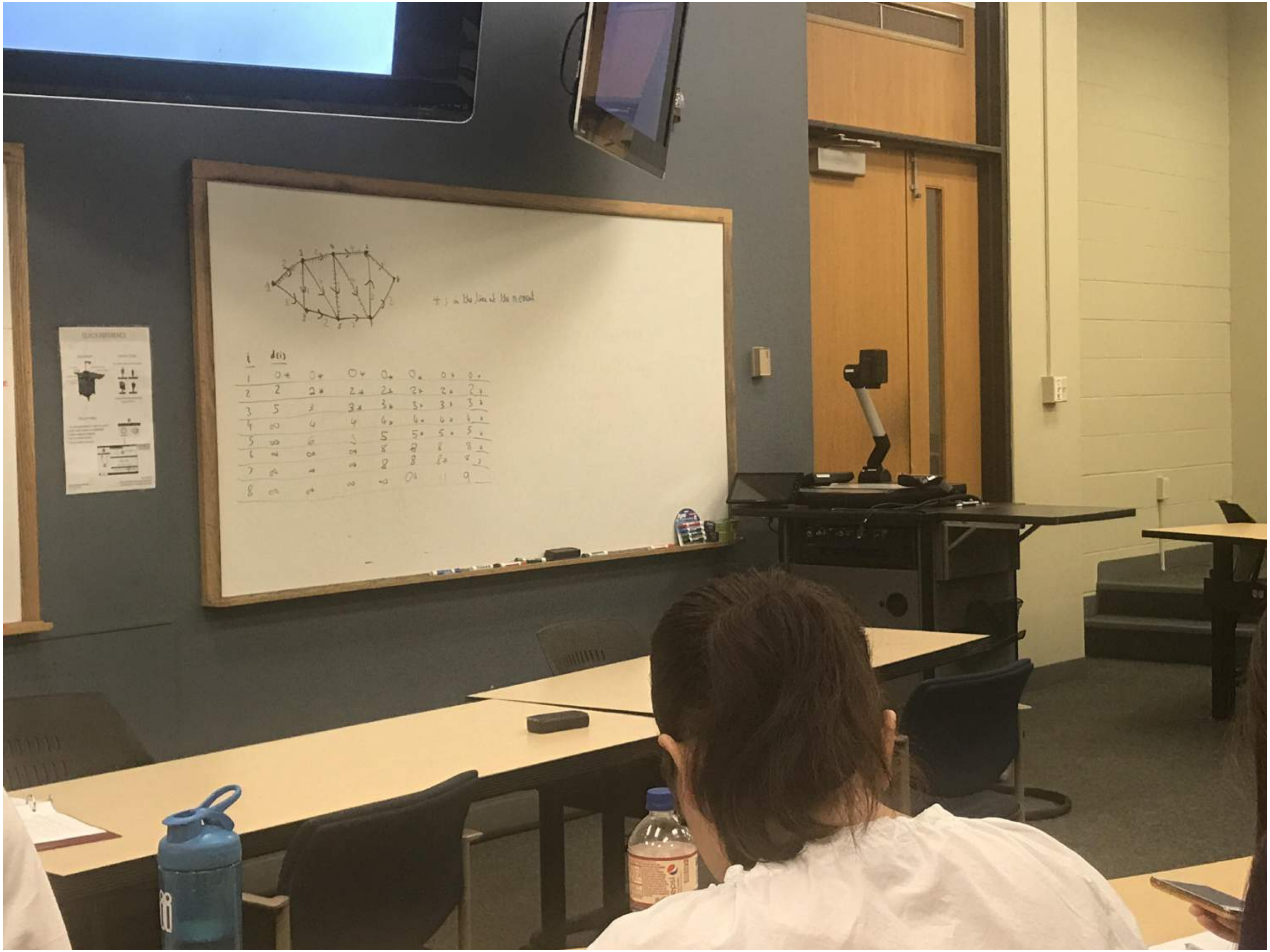

S_{k-1}
$d(i) = \min_{j \in S_{k-1}} d(j)$
Add i : $S_k = S_{k-1} \cup \{i\}$
$d(j)$ = min length path $1 — j$
$j \notin S_{k-1}$
$S_1 = \{1\}$
$d(k) = \infty$
Dijskra produces $d(1)=0, d(2), \ldots, d(n)$
$P = \{x_0 = 1, x_2, \ldots, x_m = i\}$ be a path from 1 to i
Want to show $l(P) \geq d(i)$.
Suppose i is added at round k.
x_0 x_1 x_2 x_l x_{l+1} $i = x_m$
$\in S_{k-1}$ $\in S_{k-1}$ $\in S_{k-1}$ $\in S_{k-1}$ $\notin S_{k-1}$ $\notin S_{k-1}$
$l(P) \geq l(P[1, x_{l+1}])$
because arc lengths are non-negative
$\geq d(x_{l+1})$
$\geq d(i)$



Instead of assuming $\ell(e) \geq 0, \forall e \in E$ and $\ell(P) = \ell_{rg}(P) = \sum_{e \in P} \ell(e)$

assume

$$P = (P_1, P_2) \Rightarrow \ell(P_1) \leq \ell(P).$$

Dijkstra proof is still valid

(1) Time dependent path length.

If $e = (x, y)$ then we have $a_x, b_x \geq 0$

such that if a path arrives at x at time t

then the length of e is $a_x + b_x t$

If $P = (x_0, x_1, x_2, \ldots x_m)$

$P_i = (x_0, x_1 \ldots x_i)$

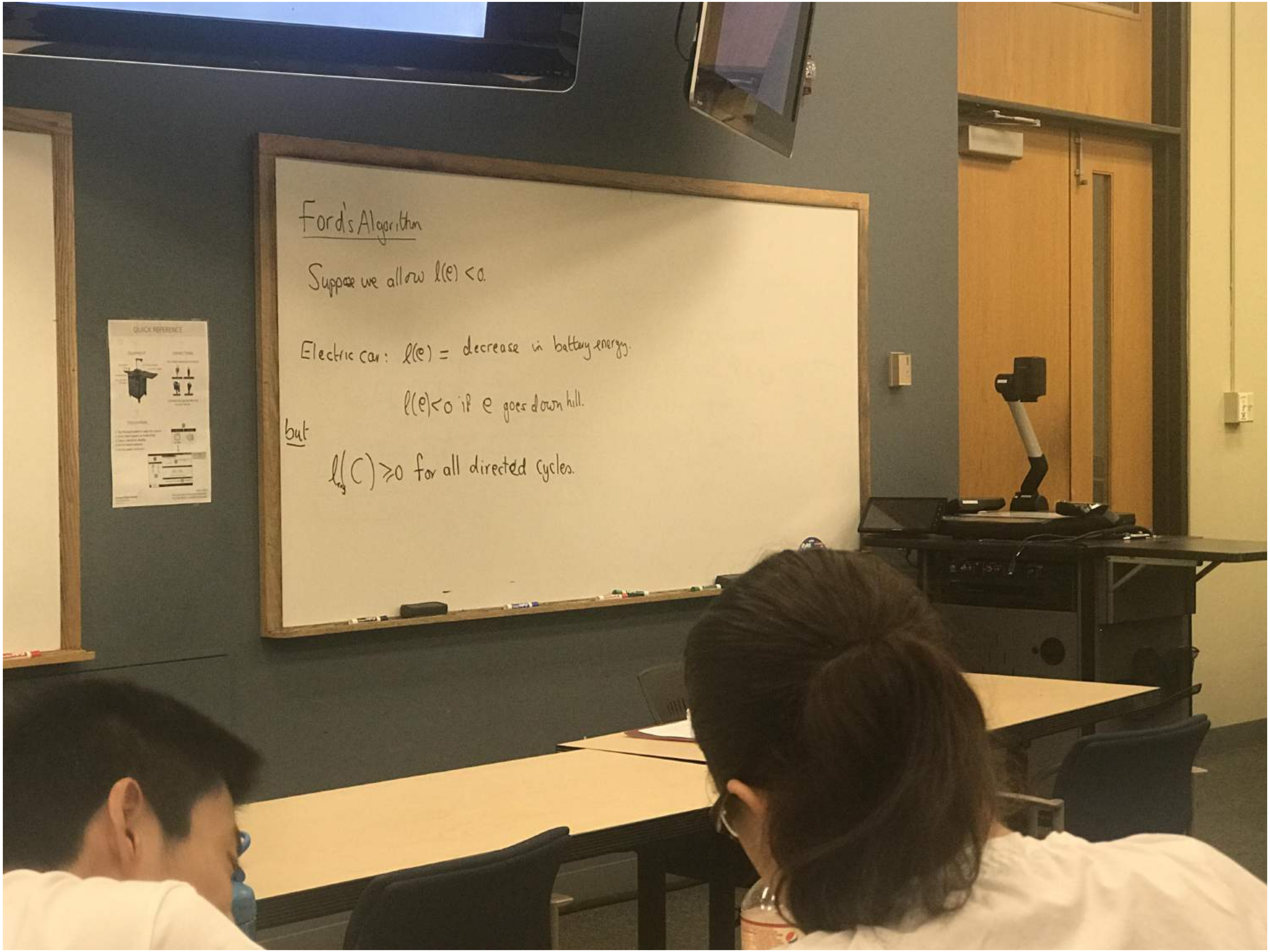
$\ell(P_{i+1}) = \ell(P_i) + a_{x_i} + b_{x_i} \ell(P_i)$

(II) Avoid S: penalty $f(k)$ for visiting S, k times.

$$\ell(P) = \ell_{reg}(P) + f(|P \cap S|) \quad \& \quad \ell(e) \geq 0, \ \forall e$$

(III) Visit S in some fixed order.

$$\ell(P) = \begin{cases} \ell_{reg}(P) & : \text{visits } S \text{ in order} \\ \infty & : \text{otherwise} \end{cases}$$


Ford's Algorithm
Suppose we allow l(e) < 0.
Electric car: l(e) = decrease in battery energy.
l(e) < 0 if e goes down hill.
but
l(C) ≥ 0 for all directed cycles.
QUICK REFERENCE